
Mathematics
*Standards of
Learning*

for
Virginia
Public
Schools

August 2023



Board of Education
Commonwealth of Virginia

Mathematics *Standards of Learning* **for** **Virginia** **Public Schools**

**Adopted in August 2023 by the
Board of Education**

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Virginia 2023 Mathematics *Standards of Learning*

Guiding Principles

In today’s data-driven world, mastery of mathematical knowledge and skills is foundational for success. Virginia’s robust and growing economy runs on workers with advanced mathematical skills. By setting clear, rigorous learning standards that are aligned to the needs of the world beyond high school, we are centering our K-12 education system on excellence. *How* we approach the implementation of these standards is just as important as defining the standards. The following Guiding Principles will help shape the policies and practices that will ensure that every Virginia student graduates high school with the mathematical knowledge and skills needed to be ready for college, career, and life.

Raise the Floor; Remove the Ceiling

Algebra is the gateway to higher education and promising careers. Therefore, every student in Virginia must continue their mathematics studies beyond Algebra before they graduate. We must also ensure there are no limits to how far or fast students can go in their mathematical studies. School divisions must clarify the pathways for the study of mathematics beyond Algebra and ensure that families are part of the decision-making process around which pathways best meet student needs and goals. The study of mathematics in Virginia will not be a “one size fits all” system. Due to the growing number of options for accessing advanced courses through dual and concurrent enrollment opportunities, Advanced Placement, Cambridge, and International Baccalaureate, there should be no limit to how far a student can progress in their mathematics journey by the time they leave high school.

All students will have the support and extra time to be able to access these rigorous pathways. The Department will partner with school divisions to ensure additional time is available for all students to access these courses. The Department will partner with school divisions to ensure extra time is available for all students to have supports such as tutoring and intervention.

Ensure Every Student Builds Strong Mathematics Foundational Skills

Students must possess strong foundational skills while also being able to apply these to real-world situations. Foundational skills, such as addition, subtraction, multiplication, division, fractions, and percentages are essential building blocks for upper-level mathematics.

What makes our foundation skill building best in class:

- Our kindergarten and first grade students are working with repeating patterns and counting by 5s and 10s while many other states are not doing this until much later. We are getting the foundations done much earlier.
- Our kindergarten and first grade students are grouping collections of objects in sets of tens and ones to strengthen their conceptual understanding of place value. Many other states do not include this in standards for kindergarten and first grade.

What makes our accelerated pathways the best in class:

- Our students are learning advanced algebraic models in middle school while in most states students are not exposed to this until high school.
- Our standards have students addressing greatest common factor in grade 4 and prime factorization of numbers and least common multiples in grade 5. Many other states do not start this work until middle school.

Master Critical Content

No student should move on to a new concept nor a new course without mastery of the prerequisite material and skills. Schools need to build processes that allow student mastery to drive how time is used rather than schedules dictating student coursework. This means rethinking the development of personalized mathematics pathways which ensure students are in courses that are challenging to them, yet manageable, as well as encouraging students who have mastered prerequisite material to enroll in more advanced courses and content beyond their grade level.

Schools, supported by their divisions and the Virginia Department of Education (VDOE), should prioritize meeting students where they are and supporting their individual academic needs—whether spending more time on mastering a difficult topic, filling a knowledge gap, or providing access to enrichment programs.

Integrate Mathematics Across All Content Areas

Mathematics appears in all content areas including a balanced equation in Chemistry, a measure in music, or a historic timeline in social studies. The application of mathematics in other content areas provides an opportunity for students to think critically and problem solve as students navigate complex data sets, design principles, and use technology in and across other content areas. Students will have the opportunity to learn mathematics in all content areas and be more informed citizens who see the connections between mathematics and the world around them.

Prepare Teachers to Teach Mathematics Accurately and Effectively

To have great mathematic students, we must have great mathematic teachers. Our educators will receive training that helps them convey content accurately and effectively to their students. By training teachers well, our students will master the basics of mathematics, deepen their mathematical knowledge and be able to apply mathematics across content areas and to real world situations. Teachers must have strong instructional materials, be trained in how to use these materials, and be able to support struggling students as well as provide advanced opportunities to those students who learn mathematics more quickly.

Apply Mathematics to Better Use Technology

Students will learn foundational mathematical skills and master computational skills before using technology as a substitute. As students' progress in their mathematical knowledge, they will apply mathematics in a variety of experiential contexts to learn and use technology appropriately. Students will learn how technology is a tool that facilitates complex mathematical thinking, requires students to solve complex problems and allows students to simulate real-world scenarios that integrates the application of mathematical reasoning and critical thinking.

Virginia 2023 Mathematics *Standards of Learning* – Background on the Revisions and a Roadmap for Successful Implementation of the Standards

Background and Overview of the Revision Process

The *Standards of Learning* provide a framework for instructional content to raise the academic achievement of all students in Virginia and to prepare them for college and career. Pursuant to legislation from the 2000 Virginia General Assembly, the Board of Education established a seven-year cycle for review of the *Standards of Learning*. As a result, the 1995 Mathematics *Standards of Learning* were reviewed in 2001, 2009, 2016, and 2023, the results of which are contained in this document.

The 2023 *Mathematics Standards* were revised with input from parents, teachers, the business community, school administrators, representatives from higher education and state mathematics education organizations. The *Standards* set clear and rigorous academic expectations for students. Parents and families are encouraged to work with their children, their children’s teachers, and their children’s schools to help them achieve these academic standards.

The following resources were central to informing this revision and ensuring that these standards are best in class:

- National Assessment of Educational Progress (NAEP) Framework (2026);
- National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000);
- NCTM Focus in High School Mathematics: Reasoning and Sense Making (2009);
- Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) report (2020) from the American Statistical Association;
- Mathematics and College Career Readiness Standards (ACT, 2022), SAT Suite of Assessments (2014) - Mathematics Test Details.

Changes from Prior Standards

There are several notable changes from the most recent set of standards. These include the following:

- 1) *An emphasis on the mastery of basic facts* - The mathematics standards establish foundational skills at each grade level and course for which students will develop proficiency. Building automaticity with basic mathematics facts is critical to ensure that students establish a firm foundation to learn more complex mathematical concepts in upper grade levels.
- 2) *Coherence* - Content standards for mathematics must be developmentally appropriate within a grade level and vertically coherent across the grade levels and courses to ensure that students master skills at each grade level. The focus of the standards concurrently emphasizes conceptual understanding, procedural knowledge, and application of mathematics content to provide building blocks from grade level to grade level and course to course.
- 3) *Mastery of Skills, Understanding of Concepts, and Application of Both*: The mathematics standards foster the application of the five mathematical process goals including reasoning, communication, problem solving, connections, and representation, and set students up to recognize and see mathematics in real-world applications. These processes support students in becoming problem solvers and to be able to make mathematical connections using mathematical representations to model and interpret contextual situations. Preparing Virginia’s students to pursue higher education, to compete in a modern workforce, and to be informed citizens requires rigorous mathematical knowledge and skills. Students must gain a

basic of mathematical skills while also gaining an understanding of ideas. The approach taken in these standards ensures students can apply mathematical concepts.

- 4) *Students will have personalized mathematics pathways based on mastery and readiness-* Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement and Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The *Standards of Learning* and knowledge and skills within each strand progress in complexity throughout the grade levels and into high school course content. The *Standards* are organized by content strand and arranged in a numeric order, however local curricula and pacing guides should determine the instructional sequence of the content that is best suited to meet the needs and goals of the student. The standards reinforce foundational mathematics skills and facts to ensure that students can accelerate at their own pace after they have mastered a concept or skill. These standards are designed to ensure mastery from grade to grade, providing an opportunity for students to demonstrate competency before learning the next skill.

The Virginia Mathematics *Standards of Learning* are built to ensure that our standards:

- 1) Are best in class, and are some of the most challenging in the nation as they are benchmarked against NAEP, ACT, and SAT;
- 2) Reinforce foundational mathematics skills and facts to ensure that students can accelerate at their own pace;
- 3) Ensure students can apply mathematical concepts in their career and college pathways; and
- 4) Are designed to ensure mastery from grade to grade and provide an opportunity for students to demonstrate competency before the next skill.

Implementation

These standards are best in class. They are benchmarked against the National Assessment for Educational Progress (NAEP), the ACT and SAT and are therefore among the most challenging mathematics standards in the nation. Successful implementation of these rigorous standards will ensure that every Virginia high school graduate has the mathematical skills, knowledge and competencies to pursue higher education, to compete in a modern workforce, and to be informed citizens.

To ensure these standards are implemented successfully, the following actions should be taken:

VDOE will:

- 1) Implement the Standards of Quality by providing opportunities for students to take Algebra and other advanced coursework in middle school;
- 2) Provide instructional guides to school divisions on how to best implement the 2023 Mathematics *Standards of Learning* to ensure a seamless transition to more rigorous expectations;
- 3) Partner with educators in regional hubs across the Commonwealth to network and collaboratively support teachers with implementation;
- 4) Ensure that teachers are equipped with high quality instructional materials and tutoring resources be able to effectively teach the new standards;
- 5) Become shoulder to shoulder partners with School Divisions to develop the capacity of teachers, Central Office leaders, and principals through professional learning opportunities in virtual, in person, and year-round opportunities;
- 6) Provide differentiated supports to school divisions who are accredited with conditions in partnership with the Office of School Quality and the Office of Student Supports;

- 7) Create regional supports and professional learning for teachers to include scaffolds and supports for English Language Learners, Gifted and Talented, and Special Education students; and
- 8) Design parent resources and supports to increase parent engagement in their students' education and mathematics achievement.

The State Board of Education will:

- 1) Review the graduation requirements to ensure students have four years of credit-bearing mathematics experiences;
- 2) Set the definition of mastery (also known as proficiency) at every grade level so that students are on track to be prepared to meet to the needs of higher education, the military, and the workplace. This proficiency cut score on assessments will be benchmarked to NAEP and the most rigorous state definitions of proficiency;
- 3) Design a state accountability system so that:
 - a) parents, teachers, policymakers have a clear picture of how every school is or is not ensuring every child is making progress to and attaining mastery of the standards; and
 - b) all students not making progress to and attaining mastery of the standards benefit from targeted supports and interventions.

Local School Divisions will:

- 1) Provide teachers with resources, training, and support to effectively implement the 2023 *Mathematics Standards of Learning*;
- 2) Ensure students, parents, and teachers have timely information on student growth and mastery of mathematics content;
- 3) Provide tutoring and interventions as just in time supports for those students struggling to reach grade level mastery;
- 4) Clearly communicate the pathways available to students within their middle school and help ready students to pursue accelerated pathways in high school; and
- 5) Highlight and increase the number of enrichment opportunities available to students through dual enrollment, summer camps, and higher education programming.

Mathematics *Standards of Learning* for Virginia Public Schools

2023 Kindergarten

The Kindergarten *Standards* place emphasis on developing the concept of number by counting, recognizing, representing, and comparing quantities; recognizing and describing with fluency part-whole relationships for numbers up to 5; as well as modeling and solving addition and subtraction problems within 10. Students will recognize, describe, and create simple repeating patterns; and recognize and describe plane figures according to their characteristics; and construct plane figures using a variety of materials. Students will investigate measurement of length, height, weight, volume, and time using direct comparisons. Students will engage with the data cycle using object graphs and picture graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

K.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 100.

Students will demonstrate the following Knowledge and Skills:

- a) Use one-to-one correspondence to determine how many are in a given set containing 30 or fewer concrete objects (e.g., cubes, pennies, balls), and describe the last number named as the total number of objects counted.
- b) Recognize and explain that the number of objects remains the same regardless of the arrangement or the order in which the objects are counted.
- c) Represent forward counting by ones using a variety of tools, including five-frames, ten-frames, and number paths (a prelude to number lines).
- d) Count forward orally by ones from 0 to 100.
- e) Count forward orally by ones, within 100, starting at any given number.
- f) Count backward orally by ones when given any number between 1 and 20.
- g) State the number after, without counting, when given any number between 0 and 30.
- h) State the number before, without counting, when given any number between 1 and 20.
- i) Use objects, drawings, words, or numbers to compose and decompose numbers 11-19 into a ten and some ones.

- j) Group a collection of up to 100 objects (e.g., counters, pennies, cubes) into sets of ten and count by tens to determine the total (e.g., there are 3 groups of ten and 6 leftovers, 36 total objects).

K.NS.2 The student will identify, represent, and compare quantities up to 30.

Students will demonstrate the following Knowledge and Skills:

- a) Read, write, and identify the numerals 0 through 30.
- b) Construct a set of objects that corresponds to a given numeral within 30, including an empty set.
- c) Determine and write the numeral that corresponds to the total number of objects in a given set of 30 or fewer concrete objects or pictorial models.
- d) Given a set of up to 30 objects, construct another set which has more, fewer, or the same number of objects using concrete or pictorial models.
- e) Given a numeral up to 30, construct a set which has more, fewer, or the same number of objects using concrete or pictorial models.
- f) Compare two sets containing up to 30 concrete objects or pictorial models, using the terms *more*, *fewer*, or the *same as (equal to)*.
- g) Compare numbers up to 30, to the benchmarks of 5 and 10 using various models (e.g., five frames, ten frames, number paths [a prelude to number lines], beaded racks, hands) using the terms *greater than*, *less than*, or the *same as (equal to)*.

Computation and Estimation

K.CE.1 The student will model and solve single-step contextual problems using addition and subtraction with whole numbers within 10.

Students will demonstrate the following Knowledge and Skills:

- a) Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 5 in multiple ways.
- b) Recognize and describe with fluency part-part-whole relationships for numbers up to 5 in a variety of configurations.
- c) Model and identify the number that makes 5 when added to a given number less than or equal to 5.
- d) Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 10 in multiple ways.
- e) Model and identify the number that makes 10 when added to a given number less than or equal to 10.
- f) Model and solve single-step contextual problems (join, separate, and part-part-whole) using 10 or fewer concrete objects.

Measurement and Geometry

K.MG.1 The student will reason mathematically by making direct comparisons between two objects or events using the attributes of length, height, weight, volume, and time.

Students will demonstrate the following Knowledge and Skills:

- a) Use direct comparisons to compare, describe, and justify the:
 - i) lengths of two objects using the terms longer or shorter;

- ii) heights of two objects using the terms taller or shorter;
- iii) weights of two objects using the terms heavier or lighter;
- iv) volumes of two containers using the terms more or less; and
- v) amount of time spent on two events using the terms longer or shorter.

K.MG.2 The student will identify, describe, name, compare, and construct plane figures (circles, triangles, squares, and rectangles).

Students will demonstrate the following Knowledge and Skills:

- a) Identify and name concrete and pictorial representations of circles, triangles, squares, and rectangles regardless of their orientation in space.
- b) Describe triangles, squares, and rectangles to include the number of sides and number of vertices.
- c) Describe a circle using terms such as *round* and *curved*.
- d) Distinguish between examples and nonexamples of identified plane figures (circles, triangles, squares, and rectangles).
- e) Compare and contrast two plane figures using characteristics to describe similarities and differences.
- f) Construct plane figures (circles, triangles, squares, and rectangles) using a variety of materials (e.g., straws, sticks, pipe cleaners).

K.MG.3 The student will describe the units of time represented in a calendar.

Students will demonstrate the following Knowledge and Skills:

- a) Identify a calendar as a tool used to measure time.
- b) Name the days of the week and state that there are seven days in one week.
- c) Determine the day before and after a given day (e.g., yesterday, today, tomorrow).
- d) Name the twelve months of the year and state that there are twelve months in one year.
- e) Distinguish between days of the week and months of the year.

Probability and Statistics

K.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs and picture graphs.

Students will demonstrate the following Knowledge and Skills:

- a) Sort and classify concrete objects into appropriate subsets (categories) based on one attribute (e.g., size, shape, color, thickness).
- b) Describe and label attributes (e.g., size, color, shape) of a set of objects (e.g., coins, counters, buttons) that has been sorted.
- c) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than four categories).
- d) Determine the data needed to answer a posed question, and collect the data using various methods (e.g., counting objects, drawing pictures).
- e) Organize and represent a data set (vertically or horizontally) by sorting concrete objects into organized groups to form a simple object graph.

- f) Organize and represent a data set (vertically or horizontally) using pictures to form a simple picture graph.
- g) Analyze data represented in object graphs and picture graphs and communicate results:
 - i) ask and answer questions about the data represented in object graphs and picture graphs (e.g., how many in each category, which categories have the greatest, least, or the same amount of data); and
 - ii) draw conclusions about the data and make predictions based on the data.

Patterns, Functions, and Algebra

K.PFA.1 The student will identify, describe, extend, and create simple repeating patterns using various representations.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and describe the core found in repeating patterns.
- b) Extend a repeating pattern by adding at least two complete repetitions of the core to the pattern.
- c) Create and describe a repeating pattern using objects, colors, sounds, movements, or pictures.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 1

The Grade 1 *Standards* place emphasis on counting, representing, comparing, and ordering sets of objects up to 120; recognizing, describing, and transferring repeating and growing patterns; and analyzing and sorting plane figures. Students' understanding of number is expanded through recognizing and describing part-part-whole relationships for numbers up to 10, recalling addition and subtraction facts to 10 with automaticity, as well as solving problems using addition and subtraction within 20. Students will begin the study of fractions by solving problems that involve partitioning models into two and four equal-sized parts. Students will use nonstandard units to measure and compare objects by length, weight, and volume. Students will engage with the data cycle using object graphs, picture graphs, and tables. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

1.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 120.

Students will demonstrate the following Knowledge and Skills:

- a) Count forward orally by ones from 0 to 120 starting at any number between 0 and 120.
- b) Count backward orally by ones when given any number between 1 and 30.
- c) Represent forward counting patterns when counting by groups of 5 and groups of 10 up to 120 using a variety of tools (e.g., objects, coins, 120 chart).
- d) Represent forward counting patterns when counting by groups of 2 up to at least 30 using a variety of tools (e.g., beaded number strings, number paths [a prelude to number lines], 120 chart).
- e) Group a collection of up to 120 objects into tens and ones, and count to determine the total (e.g., 5 groups of ten and 6 ones is equal to 56 total objects).
- f) Identify a penny, nickel, and dime by their attributes and describe the number of pennies equivalent to a nickel and a dime.
- g) Count by ones, fives, or tens to determine the value of a collection of like coins (pennies, nickels, or dimes), whose total value is 100 cents or less.

1.NS.2 The student will represent, compare, and order quantities up to 120.

Students will demonstrate the following Knowledge and Skills:

- a) Read and write numerals 0-120 in sequence and out of sequence.
- b) Estimate the number of objects (up to 120) in a given collection and justify the reasonableness of an answer.
- c) Create a concrete or pictorial representation of a number using tens and ones and write the corresponding numeral up to 120 (e.g., 47 can be represented as 47 ones or it can be grouped into 4 tens with 7 ones left over).
- d) Describe the number of groups of tens and ones when given a two-digit number and justify reasoning.
- e) Compare two numbers between 0 and 120 represented pictorially or with concrete objects using the terms *greater than*, *less than*, or *equal to*.
- f) Order three sets, each set containing up to 120 objects, from least to greatest, and greatest to least.

1.NS.3 The student will use mathematical reasoning and justification to solve contextual problems that involve partitioning models into two and four equal-sized parts.

Students will demonstrate the following Knowledge and Skills:

- a) Represent equal shares of a whole with two or four sharers, when given a contextual problem.
- b) Represent and name halves and fourths of a whole, using a region/area model (e.g., pie pieces, pattern blocks, paper folding, drawings) and a set model (e.g., eggs, marbles, counters) limited to two or four items.
- c) Describe and justify how shares are equal pieces or equal parts of the whole (limited to halves, fourths) when given a contextual problem.

Computation and Estimation

1.CE.1 The student will recall with automaticity addition and subtraction facts within 10 and represent, solve, and justify solutions to single-step problems, including those in context, using addition and subtraction with whole numbers within 20.

Students will demonstrate the following Knowledge and Skills:

- a) Recognize and describe with fluency part-part-whole relationships for numbers up to 10 in a variety of configurations.
- b) Demonstrate fluency with addition and subtraction within 10 by applying reasoning strategies (e.g., count on/count back, one more/one less, doubles, make ten).
- c) Recall with automaticity addition and subtraction facts within 10.
- d) Investigate, recognize, and describe part-part-whole relationships for numbers up to 20 in a variety of configurations (e.g., beaded racks, double ten frames).
- e) Solve addition and subtraction problems within 20 using various strategies (e.g., inverse relationships: if $9 + 3 = 12$ then $12 - 3 = 9$; decomposition using known sums/differences: $9 + 7$ can be thought of as 9 decomposed into 2 and 7, then use doubles, $7 + 7 = 14$; $14 + 2 = 16$ or decompose the 7 into 1 and 6; make a ten: $1 + 9 = 10$; $10 + 6 = 16$).
- f) Represent, solve, and justify solutions to single-step addition and subtraction problems (join, separate, and part-part-whole) within 20, including those in context, using words, objects, drawings, or numbers.

- g) Determine the unknown whole number that will result in a sum or difference of 10 or 20 (e.g., $14 - \underline{\quad} = 10$ or $15 + \underline{\quad} = 20$).
- h) Identify and use (+) as a symbol for addition and (-) as a symbol for subtraction.
- i) Describe the equal symbol (=) as a balance representing an equivalent relationship between expressions on either side of the equal symbol (e.g., 6 and 1 is the same as 4 and 3; $6 + 1$ is balanced with $4 + 3$; $6 + 1 = 4 + 3$).
- j) Use concrete materials to model, identify, and justify when two expressions are not equal (e.g., $10 - 3$ is not equal to $3 + 5$).
- k) Use concrete materials to model an equation that represents the relationship of two expressions of equal value.
- l) Write an equation that could be used to represent the solution to an oral, written, or picture problem.

Measurement and Geometry

1.MG.1 The student will reason mathematically using nonstandard units to measure and compare objects by length, weight, and volume.

Students will demonstrate the following Knowledge and Skills:

- a) Use nonstandard units to measure the:
 - i) lengths of two objects (units laid end to end with no gaps or overlaps) and compare the measurements using the terms longer/shorter, taller/shorter, or the same as;
 - ii) weights of two objects (using a balance scale or a pan scale) and compare the measurements using the terms lighter, heavier, or the same as; and
 - iii) volumes of two containers and compare the measurements using the terms more, less, or the same as.
- b) Measure the length, weight, or volume of the same object or container with two different units and describe how and why the measurements differ.

1.MG.2 The student will describe, sort, draw, and name plane figures (circles, triangles, squares, and rectangles), and compose larger plane figures by combining simple plane figures.

Students will demonstrate the following Knowledge and Skills:

- a) Describe triangles, squares, and rectangles using the terms sides, vertices, and angles. Describe a circle using terms such as *round* and *curved*.
- b) Sort plane figures based on their characteristics (e.g., number of sides, vertices, angles, curved).
- c) Draw and name the plane figure (circle, square, rectangle, triangle) when given information about the number of sides, vertices, and angles.
- d) Identify, name, and describe representations of circles, squares, rectangles, and triangles, regardless of orientation, in different environments and explain reasoning.
- e) Recognize and name the angles found in rectangles and squares as right angles.
- f) Compose larger plane figures by combining two or three simple plane figures (triangles, squares, and/or rectangles).

1.MG.3 The student will demonstrate an understanding of the concept of passage of time (to the nearest hour and half-hour) and the calendar.

Students will demonstrate the following Knowledge and Skills:

- a) Identify different tools to measure time including clocks (analog and digital) and calendar.
- b) Describe the units of time represented on a clock as minutes and hours.
- c) Tell time to the hour and half-hour, using analog and digital clocks.
- d) Describe the location of the hour hand relative to time to the hour and half-hour on an analog clock.
- e) Describe the location of the minute hand relative to time to the hour and half-hour on an analog clock.
- f) Match the time shown on a digital clock to an analog clock to the hour and half-hour.
- g) Identify specific days/dates on a calendar (e.g., What date is Saturday? How many Fridays are in October?).
- h) Use ordinal numbers first through tenth to describe the relative position of specific days/dates (e.g., What is the first Monday in October? What day of the week is May 6th?).
- i) Determine the day/date before and after a given day/date (e.g., Today is the 8th, so yesterday was the ?), and a date that is a specific number of days/weeks in the past or future (e.g., Tim's birthday is in 10 days, what will be the date of his birthday?).

Probability and Statistics

1.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs, picture graphs, and tables.

Students will demonstrate the following Knowledge and Skills:

- a) Sort and classify concrete objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, and/or thickness (e.g., sort a set of objects that are both red and thick).
- b) Describe and label attributes of a set of objects that has been sorted.
- c) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than four categories).
- d) Determine the data needed to answer a posed question and collect the data using various methods (e.g., counting objects, drawing pictures, tallying).
- e) Organize and represent a data set by sorting the collected data using various methods (e.g., tallying, T-charts).
- f) Represent a data set (vertically or horizontally) using object graphs, picture graphs, and tables.
- g) Analyze data represented in object graphs, picture graphs, and tables and communicate results:
 - i) ask and answer questions about the data represented in object graphs, picture graphs, and tables (e.g., total number of data points represented, how many in each category, how many more or less are in one category than another); and
 - ii) draw conclusions about the data and make predictions based on the data.

Patterns, Functions, and Algebra

1.PFA.1 The student will identify, describe, extend, create, and transfer repeating patterns and increasing patterns using various representations.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and describe repeating and increasing patterns.
- b) Analyze a repeating or increasing pattern and generalize the change to extend the pattern using objects, colors, movements, pictures, or geometric figures.
- c) Create a repeating or increasing pattern using objects, pictures, movements, colors, or geometric figures.
- d) Transfer a repeating or increasing pattern from one form to another.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 2

The Grade 2 *Standards* extend the study of number and spatial sense to include three-digit whole numbers and solid geometric figures. Students will demonstrate fluency and recall with automaticity addition and subtraction facts within 20. Students will also solve single-step and multistep problems, including those in context, involving addition or subtraction of whole numbers where addends or minuends do not exceed 100. Students will begin to use U.S. Customary units to measure length, weight, and liquid volume to the nearest whole unit. Students will engage with the data cycle using pictographs and bar graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

2.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 200.

Students will demonstrate the following Knowledge and Skills:

- a) Represent forward counting patterns when counting by groups of 2 up to at least 50, starting at various multiples of 2 and using a variety of tools (e.g., objects, number lines, hundreds charts).
- b) Represent forward counting patterns created when counting by groups of 5s, 10s, and 25s starting at various multiples up to at least 200 using a variety of tools (e.g., objects, number lines, hundreds charts).
- c) Describe and use patterns in skip counting by multiples of 2 (to at least 50), and multiples of 5, 10, and 25 (to at least 200) to justify the next number in the counting sequence.
- d) Represent forward counting patterns when counting by groups of 100 up to at least 1,000 starting at 0 using a variety of tools (e.g., objects, number lines, calculators, one thousand charts).
- e) Represent backward counting patterns when counting by groups of 10 from 200 or less using a variety of tools including objects, number lines, calculators, and hundreds charts.
- f) Describe and use patterns in skip counting backwards by 10s (from at least 200) to justify the next number in the counting sequence.

- g) Choose a reasonable estimate up to 1,000 when given a contextual problem (e.g., What would be the best estimate for the number of students in our school – 5, 50, or 500?).
- h) Represent even numbers (up to 50) with concrete objects, using two equal groups or two equal addends.
- i) Represent odd numbers (up to 50) with concrete objects, using two equal groups with one leftover or two equal addends plus 1.
- j) Determine whether a number (up to 50) is even or odd using concrete objects and justify reasoning (e.g., dividing collections of objects into two equal groups, pairing objects).

2.NS.2 The student will demonstrate an understanding of the ten-to-one relationships of the base 10 number system to represent, compare, and order whole numbers up to 999.

Students will demonstrate the following Knowledge and Skills:

- a) Write the three-digit whole number represented by a given model (e.g., concrete objects, pictures of base 10 blocks).
- b) Read, write, and represent three-digit numbers in standard form, expanded form, and word form, using concrete or pictorial representations.
- c) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place (ones, tens, hundreds) and value of each digit in a three-digit whole number (e.g., in 352, the 5 represents 5 tens and its value is 50).
- d) Investigate and explain the ten-to-one relationships among ones, tens, and hundreds, using models.
- e) Compose and decompose whole numbers up to 200 by making connections between a variety of models (e.g., base 10 blocks, place value cards, presented orally, in expanded or standard form) and counting strategies (e.g., 156 can be 1 hundred, 5 tens, 6 ones; 1 hundred, 4 tens, 16 ones; 15 tens, 6 ones).
- f) Plot and justify the position of a given number up to 100 on a number line with pre-marked benchmarks of 1s, 2s, 5s, 10s, or 25s.
- g) Compare two whole numbers, each 999 or less, represented concretely, pictorially, or symbolically, using words (greater than, less than, or equal to) and symbols ($>$, $<$, or $=$). Justify reasoning orally, in writing, or with a model.
- h) Order up to three whole numbers, each 999 or less, represented concretely, pictorially, or symbolically from least to greatest and greatest to least.

2.NS.3 The student will use mathematical reasoning and justification to solve contextual problems that involve partitioning models into equal-sized parts (halves, fourths, eighths, thirds, and sixths).

Students will demonstrate the following Knowledge and Skills:

- a) Model and describe fractions as representing equal-size parts of a whole.
- b) Describe the relationship between the number of fractional parts needed to make a whole and the size of the parts (i.e., as the whole is divided into more parts, each part becomes smaller).
- c) Compose the whole for a given fractional part and its value (in context) for halves, fourths, eighths, thirds, and sixths (e.g., when given $\frac{1}{4}$, determine how many pieces would be needed to make $\frac{4}{4}$).
- d) Using same-size fraction pieces, from a region/area model, count by unit fractions up to two wholes (e.g., zero one-fourths, one one-fourth, two one-fourths, three one-fourths, four one-

fourths, five one-fourths; or zero-fourths, one-fourth, two-fourths, three-fourths, four-fourths, five-fourths).

- e) Given a context, represent, name, and write fractional parts of a whole for halves, fourths, eighths, thirds, and sixths using:
 - i) region/area models (e.g., pie pieces, pattern blocks, geoboards);
 - ii) length models (e.g., paper fraction strips, fraction bars, rods, number lines); and
 - iii) set models (e.g., chips, counters, cubes).
- f) Compare unit fractions for halves, fourths, eighths, thirds, and sixths using words (greater than, less than or equal to) and symbols ($>$, $<$, $=$), with region/area and length models.

2.NS.4 The student will solve problems that involve counting and representing money amounts up to \$2.00.

Students will demonstrate the following Knowledge and Skills:

- a) Identify a quarter and its value and determine multiple ways to represent the value of a quarter using pennies, nickels, and/or dimes.
- b) Count by ones, fives, tens, and twenty-fives to determine the value of a collection of mixed coins and one-dollar bills whose total value is \$2.00 or less.
- c) Construct a set of coins and/or bills to total a given amount of money whose value is \$2.00 or less.
- d) Represent the value of a collection of coins and one-dollar bills (limited to \$2.00 or less) using the cent (¢) and dollar ($\text{\$}$) symbols and decimal point ($.$).

Computation and Estimation

2.CE.1 The student will recall with automaticity addition and subtraction facts within 20 and estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers where addends or minuends do not exceed 100.

Students will demonstrate the following Knowledge and Skills:

- a) Apply strategies (e.g., rounding to the nearest 10, compatible numbers, other number relationships) to estimate a solution for single-step addition or subtraction problems, including those in context, where addends and minuends do not exceed 100.
- b) Apply strategies (e.g., the use of concrete and pictorial models, place value, properties of addition, the relationship between addition and subtraction) to determine the sum or difference of two whole numbers where addends or minuends do not exceed 100.
- c) Represent, solve, and justify solutions to single-step and multistep contextual problems (e.g., join, separate, part-part-whole, comparison) involving addition or subtraction of whole numbers where addends or minuends do not exceed 100.
- d) Demonstrate fluency with addition and subtraction within 20 by applying reasoning strategies (e.g., doubles, near doubles, make-a-ten, compensations, inverse relationships).
- e) Recall with automaticity addition and subtraction facts within 20.
- f) Use patterns, models, and strategies to make generalizations about the algebraic properties for fluency (e.g., $4 + 3$ is equal to $3 + 4$; $0 + 8 = 8$).
- g) Determine the missing number in an equation (number sentence) through modeling and justification with addition and subtraction within 20 (e.g., $3 + \underline{\quad} = 5$ or $\underline{\quad} + 2 = 5$; $5 - \underline{\quad} = 3$ or $5 - 2 = \underline{\quad}$).

- h) Use inverse relationships to write all related facts connected to a given addition or subtraction fact model within 20 (e.g., given a model for $3 + 4 = 7$, write $4 + 3 = 7$, $7 - 4 = 3$, and $7 - 3 = 4$).
- i) Describe the not equal symbol (\neq) as representing a relationship where expressions on either side of the not equal symbol represent different values and justify reasoning.
- j) Represent and justify the relationship between values and expressions as equal or not equal using appropriate models and/or symbols (e.g., $9 + 24 = 10 + 23$; $45 - 9 = 46 - 10$; $15 + 16 \neq 31 + 15$).

Measurement and Geometry

2.MG.1 The student will reason mathematically using standard units (U.S. Customary) with appropriate tools to estimate, measure, and compare objects by length, weight, and liquid volume to the nearest whole unit.

Students will demonstrate the following Knowledge and Skills:

- a) Explain the purpose of various measurement tools and how to use them appropriately by:
 - i) identifying a ruler as an instrument to measure length;
 - ii) identifying different types of scales as instruments to measure weight; and
 - iii) identifying different types of measuring cups as instruments to measure liquid volume.
- b) Use U.S. Customary units to estimate, measure, and compare the two for reasonableness:
 - i) the length of an object to the nearest inch, using a ruler;
 - ii) the weight of an object to the nearest pound, using a scale; and
 - iii) the liquid volume of a container to the nearest cup, using a measuring cup.

2.MG.2 The student will demonstrate an understanding of the concept of time to the nearest five minutes, using analog and digital clocks.

Students will demonstrate the following Knowledge and Skills:

- a) Identify the number of minutes in an hour (60 minutes) and the number of hours in a day (24 hours).
- b) Determine the unit of time (minutes, hours, days, or weeks) that is most appropriate when measuring a given activity or context and explain reasoning (e.g., Would you measure the time it takes to brush your teeth in minutes or hours?).
- c) Show, tell, and write time to the nearest five minutes, using analog and digital clocks.
- d) Match a written time (e.g., 1:35, 6:20, 9:05) to the time shown on an analog clock to the nearest five minutes.

2.MG.3 The student will identify, describe, and create plane figures (including circles, triangles, squares, and rectangles) that have at least one line of symmetry and explain its relationship with congruency.

Students will demonstrate the following Knowledge and Skills:

- a) Explore a figure using a variety of tools (e.g., paper folding, geoboards, drawings) to show and justify a line of symmetry, if one exists.
- b) Create figures with at least one line of symmetry using various concrete and pictorial representations.
- c) Describe the two resulting figures formed by a line of symmetry as being congruent (having the same shape and size).

2.MG.4 The student will describe, name, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms).

Students will demonstrate the following Knowledge and Skills:

- a) Trace faces of solid figures (cubes and rectangular prisms) to create the set of plane figures related to the solid figure.
- b) Compare and contrast models and nets (cutouts) of cubes and rectangular prisms (e.g., number and shapes of faces, edges, vertices).
- c) Given a concrete or pictorial model, name and describe the solid figure (sphere, cube, and rectangular prism) by its characteristics (e.g., number of edges, number of vertices, shapes of faces).
- d) Compare and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms) according to their characteristics (e.g., number and shapes of their faces, edges, vertices).

Probability and Statistics

2.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs.

Students will demonstrate the following Knowledge and Skills:

- a) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than six categories).
- b) Determine the data needed to answer a posed question and collect the data using various methods (e.g., voting; creating lists, tables, or charts; tallying).
- c) Organize and represent a data set using a pictograph where each symbol represents up to 2 data points. Determine and use a key to assist in the analysis of the data.
- d) Organize and represent a data set using a bar graph with a title and labeled axes (limited to 25 or fewer data points for up to six categories, and limit increments of scale to multiples of 1 or 2).
- e) Analyze data represented in pictographs and bar graphs and communicate results:
 - i) ask and answer questions about the data represented in pictographs and bar graphs (e.g., total number of data points represented, how many in each category, how many more or less are in one category than another). Pictograph keys will be limited to symbols representing 1, 2, 5, or 10 pieces of data and bar graphs will be limited to scales with increments in multiples of 1, 2, 5, or 10; and
 - ii) draw conclusions about the data and make predictions based on the data.

Patterns, Functions, and Algebra

2.PFA.1 The student will describe, extend, create, and transfer repeating and increasing patterns (limited to addition of whole numbers) using various representations.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and describe repeating and increasing patterns.
- b) Analyze a repeating or increasing pattern and generalize the change to extend the pattern using objects, pictures, and numbers.

- c) Create a repeating or increasing pattern using various representations (e.g., objects, pictures, numbers).
- d) Transfer a given repeating or increasing pattern from one form to another (e.g., objects, pictures, numbers) and explain the connection between the two patterns.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 3

The Grade 3 *Standards* place emphasis on developing an understanding of the concepts of multiplication and division. Students will recall with automaticity multiplication facts through 10×10 and the corresponding division facts and will use this knowledge to solve contextual problems with multiplication and division. Students will extend knowledge of place value to six-digit whole numbers. Concrete models and pictorial representations will be used to deepen understanding of fraction concepts, including comparing, composing, and decomposing. Students will use standard units (U.S. Customary and metric) to measure length, weight/mass, and liquid volume. Students will be introduced to solving contextual problems involving elapsed time. Properties of shapes will be explored, and students will identify polygons, combine and subdivide polygons, and name the resulting polygon(s). Students will engage with the data cycle using pictographs and bar graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results. Students will work with a variety of growing patterns using various representations.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

3.NS.1 The student will use place value understanding to read, write, and determine the place and value of each digit in a whole number, up to six digits, with and without models.

Students will demonstrate the following Knowledge and Skills:

- a) Read and write six-digit whole numbers in standard form, expanded form, and word form.
- b) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place and value of each digit in a six-digit whole number (e.g., in 165,724, the 5 represents 5 thousands and its value is 5,000).
- c) Compose, decompose, and represent numbers up to 9,999 in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, 16 ones, but also 25 tens, 6 ones), with and without models.

3.NS.2 The student will demonstrate an understanding of the base 10 system to compare and order whole numbers up to 9,999.

Students will demonstrate the following Knowledge and Skills:

- a) Compare two whole numbers, each 9,999 or less, using symbols ($>$, $<$, $=$, \neq) and/or words (*greater than*, *less than*, *equal to*, *not equal to*), with and without models.
- b) Order up to three whole numbers, each 9,999 or less, represented with and without models, from least to greatest and greatest to least.

3.NS.3 The student will use mathematical reasoning and justification to represent and compare fractions (proper and improper) and mixed numbers with denominators of 2, 3, 4, 5, 6, 8, and 10), including those in context.

Students will demonstrate the following Knowledge and Skills:

- a) Represent, name, and write a given fraction (proper or improper) or mixed number with denominators of 2, 3, 4, 5, 6, 8, and 10 using:
 - i) region/area models (e.g., pie pieces, pattern blocks, geoboards);
 - ii) length models (e.g., paper fraction strips, fraction bars, rods, number lines); and
 - iii) set models (e.g., chips, counters, cubes).
- b) Identify a fraction represented by a model as the sum of unit fractions.
- c) Use a model of a fraction greater than one to count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4} = 1 \frac{1}{4}$).
- d) Compose and decompose fractions (proper and improper) with denominators of 2, 3, 4, 5, 6, 8, and 10 in multiple ways (e.g., $\frac{7}{4} = \frac{4}{4} + \frac{3}{4}$ or $\frac{4}{6} = \frac{3}{6} + \frac{1}{6} = \frac{2}{6} + \frac{2}{6}$) with models.
- e) Compare a fraction, less than or equal to one, to the benchmarks of 0, $\frac{1}{2}$, and 1 using area/region models, length models, and without models.
- f) Compare two fractions (proper or improper) and/or mixed numbers with like numerators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{2}{3} > \frac{2}{8}$) using words (*greater than*, *less than*, *equal to*) and/or symbols ($>$, $<$, $=$), using area/region models, length models, and without models.
- g) Compare two fractions (proper or improper) and/or mixed numbers with like denominators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{3}{6} < \frac{4}{6}$) using words (*greater than*, *less than*, *equal to*) and/or symbols ($>$, $<$, $=$), using area/region models, length models, and without models.
- h) Represent equivalent fractions with denominators of 2, 3, 4, 5, 6, 8, or 10, using region/area models and length models.

3.NS.4 The student will solve problems, including those in context, that involve counting, comparing, representing, and making change for money amounts up to \$5.00.

Students will demonstrate the following Knowledge and Skills:

- a) Determine the value of a collection of bills and coins whose total is \$5.00 or less.
- b) Construct a set of bills and coins to total a given amount of money whose value is \$5.00 or less.
- c) Compare the values of two sets of coins or two sets of bills and coins, up to \$5.00, with words (*greater than*, *less than*, *equal to*) and/or symbols ($>$, $<$, $=$) using concrete or pictorial models.
- d) Solve contextual problems to make change from \$5.00 or less by using counting on or counting back strategies with concrete or pictorial models.

Computation and Estimation

3.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers where addends and minuends do not exceed 1,000.

Students will demonstrate the following Knowledge and Skills:

- Determine and justify whether an estimate or an exact answer is appropriate when solving single-step and multistep contextual problems involving addition and subtraction, where addends and minuends do not exceed 1,000.
- Apply strategies (e.g., rounding to the nearest 10 or 100, using compatible numbers, using other number relationships) to estimate a solution for single-step or multistep addition or subtraction problems, including those in context, where addends or minuends do not exceed 1,000.
- Apply strategies (e.g., place value, properties of addition, other number relationships) and algorithms, including the standard algorithm, to determine the sum or difference of two whole numbers where addends and minuends do not exceed 1,000.
- Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal (e.g., $256 - 13 = 220 + 23$; $457 + 100 \neq 557 + 100$).
- Represent, solve, and justify solutions to single-step and multistep contextual problems involving addition and subtraction with whole numbers where addends and minuends do not exceed 1,000.

3.CE.2 The student will recall with automaticity multiplication and division facts through 10×10 ; and represent, solve, and justify solutions to single-step contextual problems using multiplication and division with whole numbers.

Students will demonstrate the following Knowledge and Skills:

- Represent multiplication and division of whole numbers through 10×10 , including in a contextual situation, using a variety of approaches and models (e.g., repeated addition/subtraction, equal-sized groups/sharing, arrays, equal jumps on a number line, using multiples to skip count).
- Use inverse relationships to write the related facts connected to a given model for multiplication and division of whole numbers through 10×10 .
- Apply strategies (e.g., place value, the properties of multiplication and/or addition) when multiplying and dividing whole numbers.
- Demonstrate fluency with multiplication facts through 10×10 by applying reasoning strategies (e.g., doubling, add-a-group, subtract-a-group, near squares, and inverse relationships).
- Represent, solve, and justify solutions to single-step contextual problems that involve multiplication and division of whole numbers through 10×10 .
- Recall with automaticity the multiplication facts through 10×10 and the corresponding division facts.
- Create an equation to represent the mathematical relationship between equivalent expressions using multiplication and/or division facts through 10×10 (e.g., $4 \times 3 = 14 - 2$, $35 \div 5 = 1 \times 7$).

Measurement and Geometry

3.MG.1 The student will reason mathematically using standard units (U.S. Customary and metric) with appropriate tools to estimate and measure objects by length, weight/mass, and liquid volume to the nearest half or whole unit.

Students will demonstrate the following Knowledge and Skills:

- a) Justify whether an estimate or an exact measurement is needed for a contextual situation and choose an appropriate unit.
- b) Estimate and measure:
 - i) length of an object to the nearest U.S. Customary unit ($\frac{1}{2}$ inch, inch, foot, yard) and metric unit (centimeter, meter);
 - ii) weight/mass of an object to the nearest U.S. Customary unit (pound) and metric unit (kilogram); and
 - iii) liquid volume to the nearest U.S. Customary unit (cup, pint, quart, gallon) and metric unit (liter).
- c) Compare estimates of length, weight/mass, or liquid volume with the actual measurements.

3.MG.2 The student will use multiple representations to estimate and solve problems, including those in context, involving area and perimeter (in both U.S. Customary and metric units).

Students will demonstrate the following Knowledge and Skills:

- a) Solve problems, including those in context, involving area:
 - i) describe and give examples of area as a measurement in contextual situations; and
 - ii) estimate and determine the area of a given surface by counting the number of square units, describe the measurement (using the number and unit) and justify the measurement.
- b) Solve problems, including those in context, involving perimeter:
 - i) describe and give examples of perimeter as a measurement in contextual situations;
 - ii) estimate and measure the distance around a polygon (with no more than six sides) to determine the perimeter and justify the measurement; and
 - iii) given the lengths of all sides of a polygon (with no more than six sides), determine its perimeter and justify the measurement.

3.MG.3 The student will demonstrate an understanding of the concept of time to the nearest minute and solve single-step contextual problems involving elapsed time in one-hour increments within a 12-hour period.

Students will demonstrate the following Knowledge and Skills:

- a) Tell and write time to the nearest minute, using analog and digital clocks.
- b) Match a written time (e.g., 4:38, 7:09, 12:51) to the time shown on analog and digital clocks to the nearest minute.
- c) Solve single-step contextual problems involving elapsed time in one-hour increments, within a 12-hour period (within a.m. or within p.m.) when given:
 - i) the starting time and the ending time, determine the amount of time that has elapsed;
 - ii) the starting time and amount of elapsed time in one-hour increments, determine the ending time; or

- iii) the ending time and the amount of elapsed time in one-hour increments, determine the starting time.

3.MG.4 The student will identify, describe, classify, compare, combine, and subdivide polygons.

Students will demonstrate the following Knowledge and Skills:

- a) Describe a polygon as a closed plane figure composed of at least three line segments that do not cross.
- b) Classify figures as polygons or not polygons and justify reasoning.
- c) Identify and describe triangles, quadrilaterals, pentagons, hexagons, and octagons in various orientations, with and without contexts.
- d) Identify and name examples of polygons (triangles, quadrilaterals, pentagons, hexagons, octagons) in the environment.
- e) Classify and compare polygons (triangles, quadrilaterals, pentagons, hexagons, octagons).
- f) Combine no more than three polygons, where each has three or four sides, and name the resulting polygon (triangles, quadrilaterals, pentagons, hexagons, octagons).
- g) Subdivide a three-sided or four-sided polygon into no more than three parts and name the resulting polygons.

Probability and Statistics

3.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs.

Students will demonstrate the following Knowledge and Skills:

- a) Formulate questions that require the collection or acquisition of data.
- b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 30 or fewer data points for no more than eight categories) using various methods (e.g., polls, observations, tallies).
- c) Organize and represent a data set using pictographs that include an appropriate title, labeled axes, and key. Each pictograph symbol should represent 1, 2, 5 or 10 data points.
- d) Organize and represent a data set using bar graphs with a title and labeled axes, with and without the use of technology tools. Determine and use an appropriate scale (increments limited to multiples of 1, 2, 5 or 10).
- e) Analyze data represented in pictographs and bar graphs, and communicate results orally and in writing:
 - i) describe the categories of data and the data as a whole (e.g., data were collected on preferred ways to cook or prepare eggs - scrambled, fried, hard boiled, and egg salad);
 - ii) identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs);
 - iii) make inferences about data represented in pictographs and bar graphs;
 - iv) use characteristics of the data to draw conclusions about the data and make predictions based on the data (e.g., it is unlikely that a third grader would like hard boiled eggs); and
 - v) solve one- and two-step addition and subtraction problems using data from pictographs and bar graphs.

Patterns, Functions, and Algebra

3.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns (limited to addition and subtraction of whole numbers), including those in context, using various representations.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and describe increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines).
- b) Analyze an increasing or decreasing pattern and generalize the change to extend the pattern or identify missing terms using various representations.
- c) Solve contextual problems that involve identifying, describing, and extending patterns.
- d) Create increasing and decreasing patterns using objects, pictures, numbers, and number lines.
- e) Investigate and explain the connection between two different representations of the same increasing or decreasing pattern.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 4

The Grade 4 *Standards* place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of decimals and fractions with like denominators. Students will recall with automaticity multiplication through 12×12 and the corresponding division facts as they become proficient in multiplying and dividing larger numbers. Students will apply knowledge of place value and the properties of addition and multiplication as strategies for solving problems. Students will identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. Students will describe and compare characteristics of plane and solid figures. Concrete models and pictorial representations will be used to solve problems involving perimeter and area, patterns, probability, and equivalence of fractions and decimals. Students will engage with the data cycle using line graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

4.NS.1 The student will use place value understanding to read, write, and identify the place and value of each digit in a nine-digit whole number.

Students will demonstrate the following Knowledge and Skills:

- a) Read nine-digit whole numbers, presented in standard form, and represent the same number in written form.
- b) Write nine-digit whole numbers in standard form when the numbers are presented orally or in written form.
- c) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place and value of each digit in a nine-digit whole number (e.g., in 568,165,724, the 8 represents 8 millions and its value is 8,000,000).

4.NS.2 The student will demonstrate an understanding of the base 10 system to compare and order whole numbers up to seven digits.

Students will demonstrate the following Knowledge and Skills:

- a) Compare two whole numbers up to seven digits each, using words (*greater than, less than, equal to, not equal to*) and/or using symbols ($>$, $<$, $=$, \neq).

- b) Order up to four whole numbers up to seven digits each, from least to greatest or greatest to least.

4.NS.3 The student will use mathematical reasoning and justification to represent, compare, and order fractions (proper, improper, and mixed numbers with denominators 12 or less), with and without models.

Students will demonstrate the following Knowledge and Skills:

- a) Compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like denominators by comparing the number of parts (numerators) using fractions with denominators of 12 or less (e.g., $\frac{1}{5} < \frac{3}{5}$). Justify comparisons orally, in writing, or with a model.*
- b) Compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like numerators and unlike denominators by comparing the size of the parts using fractions with denominators of 12 or less (e.g., $\frac{3}{8} < \frac{3}{5}$). Justify comparisons orally, in writing, or with a model.*
- c) Use benchmarks (e.g., 0, $\frac{1}{2}$, or 1) to compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like and unlike denominators of 12 or less. Justify comparisons orally, in writing, or with a model.*
- d) Compare two fractions (proper or improper) and/or mixed numbers using fractions with denominators of 12 or less, using the symbols $>$, $<$, and $=$ (e.g., $\frac{2}{3} > \frac{1}{7}$). Justify comparisons orally, in writing, or with a model.*
- e) Represent equivalent fractions with denominators of 12 or less, with and without models.*
- f) Compose and decompose fractions (proper and improper) and/or mixed numbers with denominators of 12 or less, in multiple ways, with and without models.*
- g) Represent the division of two whole numbers as a fraction given a contextual situation and a model (e.g., $\frac{3}{5}$ means the same as 3 divided by 5 or $\frac{3}{5}$ represents the amount of muffin each of five children will receive when sharing three muffins equally).

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

4.NS.4 The student will use mathematical reasoning and justification to represent, compare, and order decimals through thousandths, with and without models.

Students will demonstrate the following Knowledge and Skills:

- a) Investigate and describe the ten-to-one place value relationship for decimals through thousandths, using concrete models (e.g., place value mats/charts, decimal squares, base 10 blocks).
- b) Represent and identify decimals expressed through thousandths, using concrete, pictorial, and numerical representations.
- c) Read and write decimals expressed through thousandths, using concrete, pictorial, and numerical representations.
- d) Identify and communicate, both orally and in written form, the place and value of each digit in a decimal through thousandths (e.g., given 0.385, the 8 is in the hundredths place and has a value of 0.08).
- e) Compare using symbols ($<$, $>$, $=$) and/or words (*greater than*, *less than*, *equal to*) and order (least to greatest and greatest to least), a set of no more than four decimals expressed through

thousandths, using multiple strategies (e.g., benchmarks, place value, number lines). Justify comparisons with a model, orally, and in writing.

4.NS.5 The student will reason about the relationship between fractions and decimals (limited to halves, fourths, fifths, tenths, and hundredths) to identify and represent equivalencies.

Students will demonstrate the following Knowledge and Skills:

- Represent fractions (proper or improper) and/or mixed numbers as decimals through hundredths, using multiple representations, limited to halves, fourths, fifths, tenths, and hundredths.*
- Identify and model equivalent relationships between fractions (proper or improper) and/or mixed numbers and decimals, using halves, fourths, fifths, tenths, and hundredths.*
- Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4} = 0.25$ or $0.25 = \frac{1}{4}$; $1.25 = \frac{5}{4}$ or $1\frac{1}{4}$; $1.02 = \frac{102}{100}$ or $1\frac{2}{100}$).*

* On the state assessment, items measuring this objective are assessed without the use of a calculator.

Computation and Estimation

4.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers.

Students will demonstrate the following Knowledge and Skills:

- Determine and justify whether an estimate or an exact answer is appropriate when solving contextual problems involving addition and subtraction with whole numbers. Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*.
- Apply strategies (e.g., rounding to the nearest 100 or 1,000, using compatible numbers, other number relationships) to estimate a solution for single-step or multistep addition or subtraction problems with whole numbers, where addends or minuends do not exceed 10,000.*
- Apply strategies (e.g., place value, properties of addition, other number relationships) and algorithms, including the standard algorithm, to determine the sum or difference of two whole numbers, where addends and minuends do not exceed 10,000.*
- Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems involving addition and subtraction with whole numbers where addends and minuends do not exceed 1,000,000.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

4.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using multiplication with whole numbers, and single-step problems, including those in context, using division with whole numbers; and recall with automaticity the multiplication facts through 12×12 and the corresponding division facts.

Students will demonstrate the following Knowledge and Skills:

- Determine and justify whether an estimate or an exact answer is appropriate when solving contextual problems involving multiplication and division of whole numbers. Refine

estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*.

- b) Recall with automaticity the multiplication facts through 12×12 and the corresponding division facts.*
- c) Create an equation using addition, subtraction, multiplication, and division to represent the relationship between equivalent mathematical expressions (e.g., $4 \times 3 = 2 \times 6$; $10 + 8 = 36 \div 2$; $12 \times 4 = 60 - 12$).
- d) Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal, using addition, subtraction, multiplication, and division (e.g., $4 \times 12 = 8 \times 6$ and $64 \div 8 \neq 8 \times 8$).
- e) Determine all factor pairs for a whole number 1 to 100, using concrete, pictorial, and numerical representations.
- f) Determine common factors and the greatest common factor of no more than three numbers.
- g) Apply strategies (e.g., rounding, place value, properties of multiplication and/or addition) and algorithms, including the standard algorithm, to estimate and determine the product of two whole numbers when given:
 - i) a two-digit factor and a one-digit factor;*
 - ii) a three-digit factor and a one-digit factor;* or
 - iii) a two-digit factor and a two-digit factor.*
- h) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems that involve multiplication with whole numbers.
- i) Apply strategies (e.g., rounding, compatible numbers, place value) and algorithms, including the standard algorithm, to estimate and determine the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend, with and without remainders.*
- j) Estimate, represent, solve, and justify solutions to single-step contextual problems involving division with whole numbers.
- k) Interpret the quotient and remainder when solving a contextual problem.

* On the state assessment, items measuring this objective are assessed without the use of a calculator.

4.CE.3 The student will estimate, represent, solve, and justify solutions to single-step problems, including those in context, using addition and subtraction of fractions (proper, improper, and mixed numbers with like denominators of 2, 3, 4, 5, 6, 8, 10, and 12), with and without models; and solve single-step contextual problems involving multiplication of a whole number (12 or less) and a unit fraction, with models.

Students will demonstrate the following Knowledge and Skills:

- a) Estimate and determine the sum or difference of two fractions (proper or improper) and/or mixed numbers, having like denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12 (e.g., $\frac{3}{8} + \frac{3}{8}$, $2\frac{1}{5} + \frac{4}{5}$, $\frac{7}{4} - \frac{5}{4}$) and simplify the resulting fraction. Addition and subtraction with fractions may include regrouping.*
- b) Estimate, represent, solve, and justify solutions to single-step contextual problems using addition and subtraction with fractions (proper or improper) and/or mixed numbers, having like denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. Addition and subtraction with fractions may include regrouping.
- c) Solve single-step contextual problems involving multiplication of a whole number, limited to 12 or less, and a unit fraction (e.g., $6 \times \frac{1}{3}$, $\frac{1}{5} \times 8$, $2 \times \frac{1}{10}$), with models.*

- d) Apply the inverse property of multiplication in models (e.g., use a visual fraction model to represent $\frac{4}{4}$ or 1 as the product of $4 \times \frac{1}{4}$).

* On the state assessment, items measuring this objective are assessed without the use of a calculator.

4.CE.4 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of decimals through the thousandths, with and without models.

Students will demonstrate the following Knowledge and Skills:

- a) Apply strategies (e.g., rounding to the nearest whole number, using compatible numbers) and algorithms, including the standard algorithm, to estimate and determine the sum or difference of two decimals through the thousandths, with and without models, in which:*
- decimals do not exceed the thousandths; and
 - addends, subtrahends, and minuends are limited to four digits.
- b) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition and subtraction of decimals through the thousandths.

* On the state assessment, items measuring this objective are assessed without the use of a calculator.

Measurement and Geometry

4.MG.1 The student will reason mathematically to solve problems, including those in context, that involve length, weight/mass, and liquid volume using U.S. Customary and metric units.

Students will demonstrate the following Knowledge and Skills:

- a) Determine an appropriate unit of measure to use when measuring:
- length in both U.S. Customary (inch, foot, yard, mile) and metric units (millimeter, centimeter, meter);
 - weight/mass in both U.S. Customary (ounce, pound) and metric units (gram, kilogram); and
 - liquid volume in both U.S. Customary (cup, pint, quart, gallon) and metric units (milliliter, liter).
- b) Estimate and measure:
- length of an object to the nearest U.S. Customary unit ($\frac{1}{2}$ inch, $\frac{1}{4}$ inch, $\frac{1}{8}$ inch, foot, yard) and nearest metric unit (millimeter, centimeter, or meter);
 - weight/mass of an object to the nearest U.S. Customary unit (ounce, pound) and nearest metric unit (gram, kilogram); and
 - liquid volume to the nearest U.S. Customary unit (cup, pint, quart, gallon) and nearest metric unit (milliliter, liter).
- c) Compare estimates of length, weight/mass, or liquid volume with the actual measurements.
- d) Given the equivalent measure of one unit, solve problems, including those in context, by determining the equivalent measures within the U.S. Customary system for:
- length (inches and feet, feet and yards, inches and yards);
 - weight/mass (ounces and pounds); and
 - liquid volume (cups, pints, quarts, and gallons).

4.MG.2 The student will solve single-step and multistep contextual problems involving elapsed time (limited to hours and minutes within a 12-hour period).

Students will demonstrate the following Knowledge and Skills:

- a) Solve single-step and multistep contextual problems involving elapsed time in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.) when given:
 - i) the starting time and the ending time, determine the amount of time that has elapsed in hours and minutes;
 - ii) the starting time and amount of elapsed time in hours and minutes, determine the ending time; or
 - iii) the ending time and the amount of elapsed time in hours and minutes, determine the starting time.

4.MG.3 The student will use multiple representations to develop and use formulas to solve problems, including those in context, involving area and perimeter limited to rectangles and squares (in both U.S. Customary and metric units).

Students will demonstrate the following Knowledge and Skills:

- a) Use concrete materials and pictorial models to develop a formula for the area and perimeter of a rectangle (including a square).
- b) Determine the area and perimeter of a rectangle when given the measure of two adjacent sides (in whole number units), with and without models.
- c) Determine the area and perimeter of a square when given the measure of one side (in whole number units), with and without models.
- d) Use concrete materials and pictorial models to explore the relationship between area and perimeter of rectangles.
- e) Identify and represent rectangles with the same perimeter and different areas or with the same area and different perimeters.
- f) Solve contextual problems involving area and perimeter of rectangles and squares.

4.MG.4 The student will identify, describe, and draw points, rays, line segments, angles, and lines, including intersecting, parallel, and perpendicular lines.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices.
- b) Describe endpoints and vertices in relation to lines, line segments, rays, and angles.
- c) Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge.
- d) Identify parallel, perpendicular, and intersecting lines and line segments in plane and solid figures, including those in context.
- e) Use symbolic notation to name points, lines, line segments, rays, angles, and to describe parallel and perpendicular lines.

4.MG.5 The student will classify and describe quadrilaterals (parallelograms, rectangles, squares, rhombi, and/or trapezoids) using specific properties and attributes.

Students will demonstrate the following Knowledge and Skills:

- a) Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids through the exploration of properties and attributes.
- b) Identify and describe points, line segments, angles, and vertices in quadrilaterals.
- c) Identify and describe parallel, intersecting, perpendicular, and congruent sides in quadrilaterals.
- d) Compare, contrast, and classify quadrilaterals (parallelograms, rectangles, squares, rhombi, and/or trapezoids) based on the following properties and attributes:
 - i) parallel sides;
 - ii) perpendicular sides;
 - iii) congruence of sides; and
 - iv) number of right angles.
- e) Denote properties of quadrilaterals and identify parallel sides, congruent sides, and right angles by using geometric markings.
- f) Use symbolic notation to name line segments and angles in quadrilaterals.

4.MG.6 The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces), with and without models.

Students will demonstrate the following Knowledge and Skills:

- a) Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder).
- b) Identify and describe solid figures (cube, rectangular prism, square pyramid, and sphere) according to their characteristics (number of angles, vertices, edges, and by the number and shape of faces).
- c) Compare and contrast plane and solid figures (limited to circles, squares, triangles, rectangles, spheres, cubes, square pyramids, and rectangular prisms) according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).

Probability and Statistics

4.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line graphs.

Students will demonstrate the following Knowledge and Skills:

- a) Formulate questions that require the collection or acquisition of data.
- b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 10 or fewer data points) using various methods (e.g., observations, measurements, experiments).
- c) Organize and represent a data set using line graphs with a title and labeled axes with whole number increments, with and without the use of technology tools.
- d) Analyze data represented in line graphs and communicate results orally and in writing:
 - i) describe the characteristics of the data represented in a line graph and the data as a whole (e.g., the time period when the temperature increased the most);
 - ii) identify parts of the data that have special characteristics and explain the meaning of the greatest, the least, or the same (e.g., the highest temperature shows the warmest day);
 - iii) make inferences about data represented in line graphs;

- iv) draw conclusions about the data and make predictions based on the data to answer questions; and
- v) solve single-step and multistep addition and subtraction problems using data from line graphs.

4.PS.2 The student will model and determine the probability of an outcome of a simple event.

Students will demonstrate the following Knowledge and Skills:

- a) Describe probability as the degree of likelihood of an outcome occurring using terms such as *impossible, unlikely, equally likely, likely, and certain*.
- b) Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives (e.g., coins, two-sided counters, number cubes, spinners).
- c) Write the probability of a given simple event as a fraction between 0 and 1, where there are no more than 24 possible outcomes.
- d) Determine the likelihood of an event occurring and relate it to its whole number or fractional representation (e.g., impossible or zero; equally likely; certain or one).
- e) Create a model or contextual problem to represent a given probability.

Patterns, Functions, and Algebra

4.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns (limited to addition, subtraction, and multiplication of whole numbers), including those in context, using various representations.

Students will demonstrate the following Knowledge and Skills:

- a) Identify, describe, extend, and create increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines, input/output tables, and function machines).
- b) Analyze an increasing or decreasing single-operation numerical pattern found in lists, input/output tables, or function machines and generalize the change to identify the rule, extend the pattern, or identify missing terms.
- c) Given a rule, create increasing and decreasing patterns using numbers and input/output tables (including function machines).
- d) Solve contextual problems that involve identifying, describing, and extending increasing and decreasing patterns using single-operation input and output rules.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 5

The Grade 5 *Standards* place emphasis on solving problems, including those in context, with whole numbers, fractions, and decimals. Students will develop an understanding of prime and composite numbers, determine the prime factorization of a whole number up to 100, and solve problems using order of operations. Students will also solve problems involving area, perimeter, and volume. Students will determine the probability of an outcome by constructing a model of a sample space and using the Fundamental (Basic) Counting Principle. Students will engage with the data cycle using line plots and stem-and-leaf plots. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results. Students will be introduced to expressions with a variable to support the development of the algebraic concepts necessary for success in the middle grades.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

5.NS.1 The student will use reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compare and order sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths).

Students will demonstrate the following Knowledge and Skills:

- a) Use concrete and pictorial models to represent fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form.*
- b) Use concrete and pictorial models to represent decimals in their equivalent fraction form (thirds, eighths, and factors of 100).*
- c) Identify equivalent relationships between decimals and fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form, with and without models.*
- d) Compare (using symbols $<$, $>$, $=$) and order (least to greatest and greatest to least) a set of no more than four decimals and fractions (proper, improper) and/or mixed numbers using multiple strategies (e.g., benchmarks, place value, number lines). Justify solutions orally, in writing, or with a model.*

* On the state assessment, items measuring this objective are assessed without the use of a calculator.

5.NS.2 The student will demonstrate an understanding of prime and composite numbers, and determine the prime factorization of a whole number up to 100.

Students will demonstrate the following Knowledge and Skills:

- a) Given a whole number up to 100, create a concrete or pictorial representation to demonstrate whether the number is prime or composite, and justify reasoning.
- b) Classify, compare, and contrast whole numbers up to 100 using the characteristics prime and composite.
- c) Determine the prime factorization for a whole number up to 100.

Computation and Estimation

5.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers.

Students will demonstrate the following Knowledge and Skills:

- a) Estimate the sum, difference, product, and quotient of whole numbers in contextual problems.
- b) Represent, solve, and justify solutions to single-step and multistep contextual problems by applying strategies (e.g., estimation, properties of addition and multiplication) and algorithms, including the standard algorithm, involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders, in which:
 - i) sums, differences, and products do not exceed five digits;
 - ii) factors do not exceed two digits by three digits;
 - iii) divisors do not exceed two digits; or
 - iv) dividends do not exceed four digits.
- c) Interpret the quotient and remainder when solving a contextual problem.

5.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.

Students will demonstrate the following Knowledge and Skills:

- a) Determine the least common multiple of two numbers to find the least common denominator for two fractions.
- b) Estimate and determine the sum or difference of two fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12 (e.g., $\frac{5}{8} + \frac{1}{4}$, $\frac{4}{5} - \frac{2}{3}$, $3\frac{3}{4} + 2\frac{5}{12}$), and simplify the resulting fraction.*
- c) Estimate and solve single-step and multistep contextual problems involving addition and subtraction with fractions (proper or improper) and/or mixed numbers having like and unlike denominators, with and without models. Denominators should be limited to 2, 3, 4, 5, 6, 8, 10, and 12. Answers should be expressed in simplest form.
- d) Solve single-step contextual problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction (e.g., $9 \times \frac{2}{3}$, $8 \times \frac{3}{4}$), with models. The denominator will be a factor of the whole number and answers should be expressed in simplest form.*

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

5.CE.3 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers.

Students will demonstrate the following Knowledge and Skills:

- a) Apply estimation strategies (e.g., rounding to the nearest whole number, tenth or hundredth; compatible numbers, place value) to determine a reasonable solution for single-step and multistep contextual problems involving addition, subtraction, and multiplication of decimals, and single-step contextual problems involving division of decimals.
- b) Estimate and determine the product of two numbers using strategies and algorithms, including the standard algorithm, when given:
 - i) a two-digit factor and a one-digit factor (e.g., 2.3×4 ; 0.08×0.9 ; $.16 \times 5$);*
 - ii) a three-digit factor and a one-digit factor (e.g., 0.156×4 , 3.28×7 , 8.09×0.2);* and
 - iii) a two-digit factor and a two-digit factor (e.g., 0.85×3.7 , 14×1.6 , 9.2×3.5).*(Products will not exceed the thousandths place, and leading zeroes will not be considered when counting digits.)
- c) Estimate and determine the quotient of two numbers using strategies and algorithms, including the standard algorithm, in which:
 - i) quotients do not exceed four digits with or without a decimal point;
 - ii) quotients may include whole numbers, tenths, hundredths, or thousandths;
 - iii) divisors are limited to a single digit whole number or a decimal expressed as tenths; and
 - iv) no more than one additional zero will need to be annexed.
- d) Solve single-step and multistep contextual problems involving addition, subtraction, and multiplication of decimals by applying strategies (e.g., estimation, modeling) and algorithms, including the standard algorithm.
- e) Solve single-step contextual problems involving division with decimals by applying strategies (e.g., estimation, modeling) and algorithms, including the standard algorithm.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

5.CE.4 The student will simplify numerical expressions with whole numbers using the order of operations.

Students will demonstrate the following Knowledge and Skills:

- a) Use order of operations to simplify numerical expressions with whole numbers, limited to addition, subtraction, multiplication, and division in which:
 - i) expressions may contain no more than one set of parentheses;
 - ii) simplification will be limited to five whole numbers and four operations in any combination of addition, subtraction, multiplication, or division;
 - iii) whole numbers will be limited to two digits or less; and
 - iv) expressions should not include braces, brackets, or fraction bars.
- b) Given a whole number numerical expression involving more than one operation, describe which operation is completed first, which is second, and which is third.*

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Measurement and Geometry

5.MG.1 The student will reason mathematically to solve problems, including those in context, that involve length, mass, and liquid volume using metric units.

Students will demonstrate the following Knowledge and Skills:

- a) Determine the most appropriate unit of measure to use in a contextual problem that involves metric units:
 - i) length (millimeters, centimeters, meters, and kilometers);
 - ii) mass (grams and kilograms); and
 - iii) liquid volume (milliliters and liters).
- b) Estimate and measure to solve contextual problems that involve metric units:
 - i) length (millimeters, centimeters, and meters);
 - ii) mass (grams and kilograms); and
 - iii) liquid volume (milliliters and liters).
- c) Given the equivalent metric measure of one unit, in a contextual problem, determine the equivalent measurement within the metric system:
 - i) length (millimeters, centimeters, meters, and kilometers);
 - ii) mass (grams and kilograms); and
 - iii) liquid volume (milliliters and liters).

5.MG.2 The student will use multiple representations to solve problems, including those in context, involving perimeter, area, and volume.

Students will demonstrate the following Knowledge and Skills:

- a) Investigate and develop a formula for determining the area of a right triangle.
- b) Estimate and determine the area of a right triangle, with diagrams, when the base and the height are given in whole number units, in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 16 square inches).
- c) Describe volume as a measure of capacity and give examples of volume as a measurement in contextual situations.
- d) Investigate and develop a formula for determining the volume of rectangular prisms using concrete objects.
- e) Solve problems, including those in context, to estimate and determine the volume of a rectangular prism using concrete objects, diagrams, and formulas when the length, width, and height are given in whole number units. Record the solution with the appropriate unit of measure (e.g., 12 cubic inches).
- f) Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.
- g) Solve contextual problems that involve perimeter, area, and volume in standard units of measure.

5.MG.3 The student will classify and measure angles and triangles, and solve problems, including those in context.

Students will demonstrate the following Knowledge and Skills:

- a) Classify angles as right, acute, obtuse, or straight and justify reasoning.

- b) Classify triangles as right, acute, or obtuse and equilateral, scalene, or isosceles and justify reasoning.
- c) Identify congruent sides and right angles using geometric markings to denote properties of triangles.
- d) Compare and contrast the properties of triangles.
- e) Identify the appropriate tools (e.g., protractor, straightedge, angle ruler, available technology) to measure and draw angles.
- f) Measure right, acute, obtuse, and straight angles, using appropriate tools, and identify measures in degrees.
- g) Use models to prove that the sum of the interior angles of a triangle is 180 degrees and use the relationship to determine an unknown angle measure in a triangle.
- h) Solve addition and subtraction contextual problems to determine unknown angle measures on a diagram.

Probability and Statistics

5.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots.

Students will demonstrate the following Knowledge and Skills:

- a) Formulate questions that require the collection or acquisition of data.
- b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 30 or fewer data points) using various methods (e.g., polls, observations, measurements, experiments).
- c) Organize and represent a data set using a line plot (dot plot) with a title, labeled axes, and a key, with and without the use of technology tools. Line plots (dot plots) may contain whole numbers, fractions, or decimals.
- d) Organize and represent numerical data using a stem-and-leaf plot with a title and key, where the stems are listed in ascending order and the leaves are in ascending order, with or without commas between the leaves.
- e) Analyze data represented in line plots (dot plots) and stem-and-leaf plots and communicate results orally and in writing:
 - i) describe the characteristics of the data represented in a line plot (dot plot) and stem-and-leaf plot as a whole (e.g., the shape and spread of the data);
 - ii) make inferences about data represented in line plots (dot plots) and stem-and-leaf plots (e.g., based on a line plot (dot plot) of the number of books students in a bus line have in their backpack, every student will have from two to four books in their backpack);
 - iii) identify parts of the data that have special characteristics and explain the meaning of the greatest, the least, or the same (e.g., the stem-and-leaf plot shows that the same number of students scored in the 90s as scored in the 70s);
 - iv) draw conclusions about the data and make predictions based on the data to answer questions; and
 - v) solve single-step and multistep addition and subtraction problems using data from line plots (dot plots) and stem-and-leaf plots.

5.PS.2 The student will solve contextual problems using measures of center and the range.

Students will demonstrate the following Knowledge and Skills:

- a) Describe mean as fair share.
- b) Describe and determine the mean of a set of data values representing data from a given context as a measure of center.
- c) Describe and determine the median of a set of data values representing data from a given context as a measure of center.
- d) Describe and determine the mode of a set of data values representing data from a given context as a measure of center.
- e) Describe and determine the range of a set of data values representing data from a given context as a measure of spread.

5.PS.3 The student will determine the probability of an outcome by constructing a model of a sample space and using the Fundamental (Basic) Counting Principle.

Students will demonstrate the following Knowledge and Skills:

- a) Determine the probability of an outcome by constructing a sample space (with a total of 24 or fewer equally likely possible outcomes), using a tree diagram, list, or chart to represent and determine all possible outcomes.
- b) Determine the number of possible outcomes by using the Fundamental (Basic) Counting Principle.

Patterns, Functions, and Algebra

5.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns with whole numbers, fractions, and decimals, including those in context, using various representations.

Students will demonstrate the following Knowledge and Skills:

- a) Identify, describe, extend, and create increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines, input/output tables, function machines).
- b) Analyze an increasing or decreasing single-operation numerical pattern found in lists, input/output tables, and function machines, and generalize the change to identify the rule, extend the pattern, or identify missing terms. (Patterns will be limited to addition, subtraction, multiplication, and division of whole numbers; addition and subtraction of fractions with like denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).
- c) Solve contextual problems that involve identifying, describing, and extending increasing and decreasing patterns using single-operation input and output rules (limited to addition, subtraction, multiplication, and division of whole numbers; addition and subtraction of fractions with like denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).

5.PFA.2 The student will investigate and use variables in contextual problems.

Students will demonstrate the following Knowledge and Skills:

- a) Describe the concept of a variable (presented as a box, letter, or other symbol) as a representation of an unknown quantity.
- b) Write an equation (with a single variable that represents an unknown quantity and one operation) from a contextual situation, using addition, subtraction, multiplication, or division.

- c) Use an expression with a variable to represent a given verbal expression involving one operation (e.g., “5 more than a number” can be represented by $y + 5$).
- d) Create and write a word problem to match a given equation with a single variable and one operation.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 6

The Grade 6 *Standards* provide a transition from the emphasis placed on whole number arithmetic in the elementary grades to an introduction to algebraic thinking. The *Standards* include a focus on single-step and multistep problems involving operations with integers and positive rational numbers. Students will determine equivalency, compare, and order decimals, fractions, and percents. Students will solve problems involving area and perimeter and begin to graph in a coordinate plane. In addition, students will continue using the data cycle by applying it to circle graphs and develop concepts regarding measures of center. Students will solve linear equations in one variable, write inequality statements, and use algebraic terminology. Students will use ratios to compare two quantities and represent proportional relationships as a precursor to the development of the concept of linear functions.

The building of mathematical knowledge can be enhanced by using technology tools when used strategically and to complement further learning during grades 6-8. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and the use of instructional technology. Graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools support mathematical problem solving, reasoning, and justification, as well as application in science, engineering, business and industry, government, and practical affairs.

In grades 6-8, the acquisition of specialized mathematical vocabulary and language builds as students continue to deepen their understanding of mathematics through communication and problem solving.

Problem solving increases in complexity as students progress into middle and high school. Students continue to build upon their mathematical knowledge by participating in problem-solving activities that promote higher levels of critical thinking and analysis.

Number and Number Sense

6.NS.1 The student will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents.

Students will demonstrate the following Knowledge and Skills:

- a) Estimate and determine the percent represented by a given model (e.g., number line, picture, verbal description), including percents greater than 100% and less than 1%.*
- b) Represent and determine equivalencies among decimals (through the thousandths place) and percents incorporating the use of number lines, and concrete and pictorial models.*
- c) Represent and determine equivalencies among fractions (proper or improper) and mixed numbers that have denominators that are 12 or less or factors of 100 and percents incorporating the use of number lines, and concrete and pictorial models.*
- d) Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100 incorporating the use of number lines, and concrete and pictorial models.*
- e) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than four positive rational numbers expressed as fractions (proper or improper), mixed

numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100) with and without models. Justify solutions orally, in writing or with a model. Ordering may be in ascending or descending order.*

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

6.NS.2 The student will reason and use multiple strategies to represent, compare, and order integers.

Students will demonstrate the following Knowledge and Skills:

- Represent integers (e.g., number lines, concrete materials, pictorial models), including models derived from contextual situations, and identify an integer represented by a point on a number line.
- Compare and order integers using a number line.
- Compare integers, using mathematical symbols ($<$, $>$, $=$).
- Identify and describe the absolute value of an integer as the distance from zero on the number line.

6.NS.3 The student will recognize and represent patterns with whole number exponents and perfect squares.

Students will demonstrate the following Knowledge and Skills:

- Recognize and represent patterns with bases and exponents that are whole numbers.
- Recognize and represent patterns of perfect squares not to exceed 20^2 , by using concrete and pictorial models.
- Justify if a number between 0 and 400 is a perfect square through modeling or mathematical reasoning.
- Recognize and represent powers of 10 with whole number exponents by examining patterns in place value.

Computation and Estimation

6.CE.1 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context.

Students will demonstrate the following Knowledge and Skills:

- Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations.*
- Multiply and divide fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.*
- Investigate and explain the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one.*
- Estimate, determine, and justify the solution to single-step and multistep problems in context that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form.
- Estimate, determine, and justify the solution to single-step and multistep problems in context that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

6.CE.2 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.

Students will demonstrate the following Knowledge and Skills:

- a) Demonstrate/model addition, subtraction, multiplication, and division of integers using pictorial representations or concrete manipulatives.*
- b) Add, subtract, multiply, and divide two integers.*
- c) Simplify an expression that contains absolute value bars $||$ and an operation with two integers (e.g., $-|5 - 8|$ or $\frac{|-12|}{8}$) and represent the result on a number line.
- d) Estimate, determine, and justify the solution to one and two-step contextual problems, involving addition, subtraction, multiplication, and division with integers.

* On the state assessment, items measuring this objective are assessed without the use of a calculator.

Measurement and Geometry

6.MG.1 The student will identify the characteristics of circles and solve problems, including those in context, involving circumference and area.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and describe chord, diameter, radius, circumference, and area of a circle.
- b) Investigate and describe the relationship between:
 - i) diameter and radius;
 - ii) radius and circumference; and
 - iii) diameter and circumference.
- c) Develop an approximation for pi (3.14) by gathering data and comparing the circumference to the diameter of various circles, using concrete manipulatives or technological models.
- d) Develop the formula for circumference using the relationship between diameter, radius, and pi.
- e) Solve problems, including those in context, involving circumference and area of a circle when given the length of the diameter or radius.

6.MG.2 The student will reason mathematically to solve problems, including those in context, that involve the area and perimeter of triangles and parallelograms.

Students will demonstrate the following Knowledge and Skills:

- a) Develop the formula for determining the area of parallelograms and triangles using pictorial representations and concrete manipulatives (e.g., two-dimensional diagrams, grid paper).
- b) Solve problems, including those in context, involving the perimeter and area of triangles and parallelograms.

6.MG.3 The student will describe the characteristics of the coordinate plane and graph ordered pairs.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and label the axes, origin, and quadrants of a coordinate plane.
- b) Identify and describe the location (quadrant or the axis) of a point given as an ordered pair. Ordered pairs will be limited to coordinates expressed as integers.

- c) Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.
- d) Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.
- e) Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers.
- f) Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving contextual and mathematical problems.

6.MG.4 The student will determine congruence of segments, angles, and polygons.

Students will demonstrate the following Knowledge and Skills:

- a) Identify regular polygons.
- b) Draw lines of symmetry to divide regular polygons into two congruent parts.
- c) Determine the congruence of segments, angles, and polygons given their properties.
- d) Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

Probability and Statistics

6.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.

Students will demonstrate the following Knowledge and Skills:

- a) Formulate questions that require the collection or acquisition of data with a focus on circle graphs.
- b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).
- c) Determine the factors that will ensure that the data collected is a sample that is representative of a larger population.
- d) Organize and represent data using circle graphs, with and without the use of technology tools. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%).
- e) Analyze data represented in a circle graph by making observations and drawing conclusions.
- f) Compare data represented in a circle graph with the same data represented in other graphs, including but not limited to bar graphs, pictographs, and line plots (dot plots), and justify which graphical representation best represents the data.

6.PS.2 The student will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed.

Students will demonstrate the following Knowledge and Skills:

- a) Represent the mean of a set of data graphically as the balance point represented in a line plot (dot plot).

- b) Determine the effect on measures of center when a single value of a data set is added, removed, or changed.
- c) Observe patterns in data to identify outliers and determine their effect on mean, median, mode, or range.

Patterns, Functions, and Algebra

6.PFA.1 The student will use ratios to represent relationships between quantities, including those in context.

Students will demonstrate the following Knowledge and Skills:

- a) Represent a relationship between two quantities using ratios.
- b) Represent a relationship in context that makes a comparison by using the notations $\frac{a}{b}$, $a:b$, and a to b .
- c) Represent different comparisons within the same quantity or between different quantities (e.g., part to part, part to whole, whole to whole).
- d) Create a relationship in words for a given ratio expressed symbolically.
- e) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio.
- f) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a contextual situation.

6.PFA.2 The student will identify and represent proportional relationships between two quantities, including those in context (unit rates are limited to positive values).

Students will demonstrate the following Knowledge and Skills:

- a) Identify the unit rate of a proportional relationship represented by a table of values, a contextual situation, or a graph.
- b) Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate.
- c) Determine whether a proportional relationship exists between two quantities, when given a table of values, context, or graph.
- d) When given a contextual situation representing a proportional relationship, find the unit rate and create a table of values or a graph.
- e) Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs.

6.PFA.3 The student will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and develop examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient.
- b) Represent and solve one-step linear equations in one variable, using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).

- c) Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers.
- d) Confirm solutions to one-step linear equations in one variable using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).
- e) Write a one-step linear equation in one variable to represent a verbal situation, including those in context.
- f) Create a verbal situation in context given a one-step linear equation in one variable.

6.PFA.4 The student will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.

Students will demonstrate the following Knowledge and Skills:

- a) Given the graph of a linear inequality in one variable on a number line, represent the inequality in two equivalent ways (e.g., $x < -5$ or $-5 > x$) using symbols. Symbols include $<$, $>$, \leq , \geq .
- b) Write a linear inequality in one variable to represent a given constraint or condition in context or given a graph on a number line.
- c) Given a linear inequality in one variable, create a corresponding contextual situation or create a number line graph.
- d) Use substitution or a number line graph to justify whether a given number in a specified set makes a linear inequality in one variable true.
- e) Identify a numerical value(s) that is part of the solution set of a given inequality in one variable.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 7

The Grade 7 *Standards* continue to emphasize the foundations of Algebra. The *Standards* address the representation and comparison of rational numbers using exponents, scientific notation, and square roots. Students continue to develop proficiency in operations with rational numbers and solving problems in context by expanding their study from Grade 6. Students will build on the concept of ratios to solve problems involving proportional reasoning, which is emphasized throughout the Grade 7 *Standards*. Students will solve problems involving volume and surface area and focus on the relationships among the properties of quadrilaterals. In addition, students will continue to apply the data cycle and extend the application to histograms. Probability is investigated through comparing experimental results to theoretical expectations. Students continue to develop their understanding of solving linear equations and inequalities in one variable by applying the properties of real numbers. Students investigate proportional relationships and begin to develop a concept of slope as rate of change.

The building of mathematical knowledge can be enhanced by using technology tools when used strategically and to complement further learning during grades 6-8. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and the use of instructional technology. Graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools support mathematical problem solving, reasoning, and justification, as well as application in science, engineering, business and industry, government, and practical affairs.

In grades 6-8, the acquisition of specialized mathematical vocabulary and language builds as students continue to deepen their understanding of mathematics through communication and problem solving.

Problem solving increases in complexity as students progress into middle and high school. Students continue to build upon their mathematical knowledge by participating in problem-solving activities that promote higher levels of critical thinking and analysis.

Number and Number Sense

7.NS.1 The student will investigate and describe the concept of exponents for powers of ten and compare and order numbers greater than zero written in scientific notation.

Students will demonstrate the following Knowledge and Skills:

- a) Investigate and describe powers of 10 with negative exponents by examining patterns.
- b) Represent a power of 10 with a negative exponent in fraction and decimal form.
- c) Convert between numbers greater than 0 written in scientific notation and decimals.*
- d) Compare and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order.*

* On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.

7.NS.2 The student will reason and use multiple strategies to compare and order rational numbers.

Students will demonstrate the following Knowledge and Skills:

- a) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare (using symbols $<$, $>$, $=$) and order (a set of no more than four) rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. Ordering may be in ascending or descending order. Justify solutions orally, in writing or with a model.*

* On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.

7.NS.3 The student will recognize and describe the relationship between square roots and perfect squares.

Students will demonstrate the following Knowledge and Skills:

- a) Determine the positive square root of a perfect square from 0 to 400.*
- b) Describe the relationship between square roots and perfect squares.*

* On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.

Computation and Estimation

7.CE.1 The student will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers.

Students will demonstrate the following Knowledge and Skills:

- a) Estimate, solve, and justify solutions to contextual problems involving addition, subtraction, multiplication, and division with rational numbers expressed as integers, fractions (proper or improper), mixed numbers, and decimals. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place.

7.CE.2 The student will solve problems, including those in context, involving proportional relationships.

Students will demonstrate the following Knowledge and Skills:

- a) Given a proportional relationship between two quantities, create and use a ratio table to determine missing values.
- b) Write and solve a proportion that represents a proportional relationship between two quantities to find a missing value, including problems in context.
- c) Apply proportional reasoning to solve problems in context, including converting units of measurement, when given the conversion factor.
- d) Estimate and determine the percentage of a given whole number, including but not limited to the use of benchmark percentages.

Measurement and Geometry

7.MG.1 The student will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context.

Students will demonstrate the following Knowledge and Skills:

- a) Develop the formulas for determining the volume of right cylinders and solve problems, including those in contextual situations, using concrete objects, diagrams, and formulas.

- b) Develop the formulas for determining the surface area of rectangular prisms and right cylinders and solve problems, including those in contextual situations, using concrete objects, two-dimensional diagrams, nets, and formulas.
- c) Determine if a problem in context, involving a rectangular prism or right cylinder, represents the application of volume or surface area.
- d) Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, 2, 3, or 4, including those in contextual situations.
- e) Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{2}$ or 2, including those in contextual situations.

7.MG.2 The student will solve problems and justify relationships of similarity using proportional reasoning.

Students will demonstrate the following Knowledge and Skills:

- a) Identify corresponding congruent angles of similar quadrilaterals and triangles, through the use of geometric markings.
- b) Identify corresponding sides of similar quadrilaterals and triangles.
- c) Given two similar quadrilaterals or triangles, write similarity statements using symbols.
- d) Write proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles.
- e) Recognize and justify if two quadrilaterals or triangles are similar using the ratios of corresponding side lengths.
- f) Solve a proportion to determine a missing side length of similar quadrilaterals or triangles.
- g) Given angle measures in a quadrilateral or triangle, determine unknown angle measures in a similar quadrilateral or triangle.
- h) Apply proportional reasoning to solve problems in context including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths.

7.MG.3 The student will compare and contrast quadrilaterals based on their properties and determine unknown side lengths and angle measures of quadrilaterals.

Students will demonstrate the following Knowledge and Skills:

- a) Compare and contrast properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid:
 - i) parallel/perpendicular sides and diagonals;
 - ii) congruence of angle measures, side, and diagonal lengths; and
 - iii) lines of symmetry.
- b) Sort and classify quadrilaterals as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties:
 - i) parallel/perpendicular sides and diagonals;
 - ii) congruence of angle measures, side, and diagonal lengths; and
 - iii) lines of symmetry.
- c) Given a diagram, determine an unknown angle measure in a quadrilateral, using properties of quadrilaterals.
- d) Given a diagram, determine an unknown side length in a quadrilateral using properties of quadrilaterals.

7.MG.4 The student will apply dilations of polygons in the coordinate plane.

Students will demonstrate the following Knowledge and Skills:

- a) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been dilated. Scale factors are limited to $\frac{1}{4}$, $\frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin.
- b) Sketch the image of a dilation of a polygon limited to a scale factor of $\frac{1}{4}$, $\frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin.
- c) Identify and describe dilations in context including, but not limited to, scale drawings and graphic design.

Probability and Statistics

7.PS.1 The student will use statistical investigation to determine the probability of an event and investigate and describe the difference between the experimental and theoretical probability.

Students will demonstrate the following Knowledge and Skills:

- a) Determine the theoretical probability of an event.
- b) Given the results of a statistical investigation, determine the experimental probability of an event.
- c) Describe changes in the experimental probability as the number of trials increases.
- d) Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event.

7.PS.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on histograms.

Students will demonstrate the following Knowledge and Skills:

- a) Formulate questions that require the collection or acquisition of data with a focus on histograms.
- b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).
- c) Determine how sample size and randomness will ensure that the data collected is a sample that is representative of a larger population.
- d) Organize and represent numerical data using histograms with and without the use of technology.
- e) Investigate and explain how using different intervals could impact the representation of the data in a histogram.
- f) Compare data represented in histograms with the same data represented in other graphs, including but not limited to line plots (dot plots), circle graphs, and stem-and-leaf plots, and justify which graphical representation best represents the data.
- g) Analyze data represented in histograms by making observations and drawing conclusions. Determine how histograms reveal patterns in data that cannot be easily seen by looking at the corresponding given data set.

Patterns, Functions, and Algebra

7.PFA.1 The student will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context.

Students will demonstrate the following Knowledge and Skills:

- Determine the slope, m , as the rate of change in a proportional relationship between two quantities given a table of values, graph, or contextual situation and write an equation in the form $y = mx$ to represent the direct variation relationship. Slope may include positive or negative values (slope will be limited to positive values in a contextual situation).
- Identify and describe a line with a slope that is positive, negative, or zero (0), given a graph.
- Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, m , as rate of change. Slope may include positive or negative values.
- Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where m represents the slope as rate of change. Slope may include positive or negative values.
- Make connections between and among representations of a proportional relationship between two quantities using problems in context, tables, equations, and graphs. Slope may include positive or negative values (slope will be limited to positive values in a contextual situation).

7.PFA.2 The student will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables.

Students will demonstrate the following Knowledge and Skills:

- Use the order of operations and apply the properties of real numbers to simplify numerical expressions. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [] and absolute value bars | |. Square roots are limited to perfect squares.*
- Represent equivalent algebraic expressions in one variable using concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles).
- Simplify and generate equivalent algebraic expressions in one variable by applying the order of operations and properties of real numbers. Expressions may require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be positive or negative rational numbers.*
- Use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [] and absolute value bars | |. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression. Replacement values may be positive or negative rational numbers.

* On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.

7.PFA.3 The student will write and solve two-step linear equations in one variable, including problems in context, that require the solution of a two-step linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

- a) Represent and solve two-step linear equations in one variable using a variety of concrete materials and pictorial representations.
- b) Apply properties of real numbers and properties of equality to solve two-step linear equations in one variable. Coefficients and numeric terms will be rational.
- c) Confirm algebraic solutions to linear equations in one variable.
- d) Write a two-step linear equation in one variable to represent a verbal situation, including those in context.
- e) Create a verbal situation in context given a two-step linear equation in one variable.
- f) Solve problems in context that require the solution of a two-step linear equation.

7.PFA.4 The student will write and solve one- and two-step linear inequalities in one variable, including problems in context, that require the solution of a one- and two-step linear inequality in one variable.

Students will demonstrate the following Knowledge and Skills:

- a) Apply properties of real numbers and the addition, subtraction, multiplication, and division properties of inequality to solve one- and two-step inequalities in one variable. Coefficients and numeric terms will be rational.
- b) Investigate and explain how the solution set of a linear inequality is affected by multiplying or dividing both sides of the inequality statement by a rational number less than zero.
- c) Represent solutions to one- or two-step linear inequalities in one variable algebraically and graphically using a number line.
- d) Write one- or two-step linear inequalities in one variable to represent a verbal situation, including those in context.
- e) Create a verbal situation in context given a one or two-step linear inequality in one variable.
- f) Solve problems in context that require the solution of a one- or two-step inequality.
- g) Identify a numerical value(s) that is part of the solution set of as given one- or two-step linear inequality in one variable.
- h) Describe the differences and similarities between solving linear inequalities in one variable and linear equations in one variable.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 8

The Grade 8 *Standards* continue to build on the concepts needed for success in high school level Algebra, Geometry, and Statistics. Students will explore real numbers and the subsets of the real number system. Proportional reasoning is expounded upon as students solve a variety of problems. Students find the volume and surface area of more complex three-dimensional figures and apply transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem creating a foundation for further study of triangular relationships in Geometry. Students will represent data, both univariate and bivariate, and continue to apply the data cycle and extend the application to boxplots and scatterplots. Students build upon the algebraic concepts developed in the Grade 6 and 7 *Standards*, which include simplifying algebraic expressions, solving multistep linear equations and inequalities in one variable, and graphing linear functions. The Grade 8 *Standards* are vital to providing a solid foundation in Algebra 1.

The building of mathematical knowledge can be enhanced by using technology tools when used strategically and to complement further learning during grades 6-8. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and the use of instructional technology. Graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools support mathematical problem solving, reasoning, and justification, as well as application in science, engineering, business and industry, government, and practical affairs.

In grades 6-8, the acquisition of specialized mathematical vocabulary and language builds as students continue to deepen their understanding of mathematics through communication and problem solving.

Problem solving increases in complexity as students progress into middle and high school. Students continue to build upon their mathematical knowledge by participating in problem-solving activities that promote higher levels of critical thinking and analysis.

Number and Number Sense

8.NS.1 The student will compare and order real numbers and determine the relationships between real numbers.

Students will demonstrate the following Knowledge and Skills:

- a) Estimate and identify the two consecutive natural numbers between which the positive square root of a given number lies and justify which natural number is the better approximation. Numbers are limited to natural numbers from 1 to 400.
- b) Use rational approximations (to the nearest hundredth) of irrational numbers to compare, order, and locate values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number.
- c) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than five real numbers expressed as integers, fractions (proper or improper), decimals, mixed numbers, percents, numbers written in scientific notation, radicals, and π . Radicals may include both positive and negative square roots of values from 0 to 400. Ordering may be in ascending or descending order. Justify solutions orally, in writing or with a model.

8.NS.2 The student will investigate and describe the relationship between the subsets of the real number system.

Students will demonstrate the following Knowledge and Skills:

- a) Describe and illustrate the relationships among the subsets of the real number system by using representations (e.g., graphic organizers, number lines). Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural numbers.
- b) Classify and explain why a given number is a member of a particular subset or subsets of the real number system.
- c) Describe each subset of the set of real numbers and include examples and non-examples.

Computation and Estimation

8.CE.1 The student will estimate and apply proportional reasoning and computational procedures to solve contextual problems.

Students will demonstrate the following Knowledge and Skills:

- a) Estimate and solve contextual problems that require the computation of one discount or markup and the resulting sale price.
- b) Estimate and solve contextual problems that require the computation of the sales tax, tip and resulting total.
- c) Estimate and solve contextual problems that require the computation of the percent increase or decrease.

Measurement and Geometry

8.MG.1 The student will use the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles to determine the measure of unknown angles.

Students will demonstrate the following Knowledge and Skills:

- a) Identify and describe the relationship between pairs of angles that are vertical, adjacent, supplementary, and complementary.
- b) Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve problems, including those in context, involving the measure of unknown angles.

8.MG.2 The student will investigate and determine the surface area of square-based pyramids and the volume of cones and square-based pyramids.

Students will demonstrate the following Knowledge and Skills:

- a) Determine the surface area of square-based pyramids by using concrete objects, nets, diagrams, and formulas.
- b) Determine the volume of cones and square-based pyramids, using concrete objects, diagrams, and formulas.
- c) Examine and explain the relationship between the volume of cones and cylinders, and the volume of rectangular prisms and square based pyramids.
- d) Solve problems in context involving volume of cones and square-based pyramids and the surface area of square-based pyramids.

8.MG.3 The student will apply translations and reflections to polygons in the coordinate plane.

Students will demonstrate the following Knowledge and Skills:

- a) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated vertically, horizontally, or a combination of both.
- b) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been reflected over the x - or y -axis.
- c) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated and reflected over the x - or y -axis or reflected over the x - or y -axis and then translated.
- d) Sketch the image of a polygon that has been translated vertically, horizontally, or a combination of both.
- e) Sketch the image of a polygon that has been reflected over the x - or y -axis.
- f) Sketch the image of a polygon that has been translated and reflected over the x - or y -axis, or reflected over the x - or y -axis and then translated.
- g) Identify and describe transformations in context (e.g., tiling, fabric, wallpaper designs, art).

8.MG.4 The student will apply the Pythagorean Theorem to solve problems involving right triangles, including those in context.

Students will demonstrate the following Knowledge and Skills:

- a) Verify the Pythagorean Theorem using diagrams, concrete materials, and measurement.
- b) Determine whether a triangle is a right triangle given the measures of its three sides.
- c) Identify the parts of a right triangle (the hypotenuse and the legs) given figures in various orientations.
- d) Determine the measure of a side of a right triangle, given the measures of the other two sides.
- e) Apply the Pythagorean Theorem, and its converse, to solve problems involving right triangles in context.

8.MG.5 The student will solve area and perimeter problems involving composite plane figures, including those in context.

Students will demonstrate the following Knowledge and Skills:

- a) Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, circles, and semicircles. Determine the area of subdivisions and combine to determine the area of the composite plane figure.
- b) Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Use the attributes of the subdivisions to determine the perimeter of the composite plane figure.
- c) Apply perimeter, circumference, and area formulas to solve contextual problems involving composite plane figures.

Probability and Statistics

8.PS.1 The student will use statistical investigation to determine the probability of independent and dependent events, including those in context.

Students will demonstrate the following Knowledge and Skills:

- a) Determine whether two events are independent or dependent and explain how replacement impacts the probability.
- b) Compare and contrast the probability of independent and dependent events.

- c) Determine the probability of two independent events.
- d) Determine the probability of two dependent events.

8.PS.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on boxplots.

Students will demonstrate the following Knowledge and Skills:

- a) Formulate questions that require the collection or acquisition of data with a focus on boxplots.
- b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).
- c) Determine how statistical bias might affect whether the data collected from the sample is representative of the larger population.
- d) Organize and represent a numeric data set of no more than 20 items, using boxplots, with and without the use of technology.
- e) Identify and describe the lower extreme (minimum), upper extreme (maximum), median, upper quartile, lower quartile, range, and interquartile range given a data set, represented by a boxplot.
- f) Describe how the presence of an extreme data point (outlier) affects the shape and spread of the data distribution of a boxplot.
- g) Analyze data represented in a boxplot by making observations and drawing conclusions.
- h) Compare and analyze two data sets represented in boxplots.
- i) Given a contextual situation, justify which graphical representation (e.g., pictographs, bar graphs, line graphs, line plots/dot plots, stem-and-leaf plots, circle graphs, histograms, and boxplots) best represents the data.
- j) Identify components of graphical displays that can be misleading.

8.PS.3 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on scatterplots.

Students will demonstrate the following Knowledge and Skills:

- a) Formulate questions that require the collection or acquisition of data with a focus on scatterplots.
- b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) of no more than 20 items using various methods (e.g., observations, measurement, surveys, experiments).
- c) Organize and represent numeric bivariate data using scatterplots with and without the use of technology.
- d) Make observations about a set of data points in a scatterplot as having a positive linear relationship, a negative linear relationship, or no relationship
- e) Analyze and justify the relationship of the quantitative bivariate data represented in scatterplots.
- f) Sketch the line of best fit for data represented in a scatterplot.

Patterns, Functions, and Algebra

8.PFA.1 The student will represent, simplify, and generate equivalent algebraic expressions in one variable.

Students will demonstrate the following Knowledge and Skills:

- Represent algebraic expressions using concrete manipulatives or pictorial representations (e.g., colored chips, algebra tiles), including expressions that apply the distributive property.
- Simplify and generate equivalent algebraic expressions in one variable by applying the order of operations and properties of real numbers. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational.

8.PFA.2 The student will determine whether a given relation is a function and determine the domain and range of a function.

Students will demonstrate the following Knowledge and Skills:

- Determine whether a relation, represented by a set of ordered pairs, a table, or a graph of discrete points is a function. Sets are limited to no more than 10 ordered pairs.
- Identify the domain and range of a function represented as a set of ordered pairs, a table, or a graph of discrete points.

8.PFA.3 The student will represent and solve problems, including those in context, by using linear functions and analyzing their key characteristics (the value of the y-intercept (b) and the coordinates of the ordered pairs in graphs will be limited to integers).

Students will demonstrate the following Knowledge and Skills:

- Determine how adding a constant (b) to the equation of a proportional relationship $y = mx$ will translate the line on a graph.
- Describe key characteristics of linear functions including slope (m), y-intercept (b), and independent and dependent variables.
- Graph a linear function given a table, equation, or a situation in context.
- Create a table of values for a linear function given a graph, equation in the form of $y = mx + b$, or context.
- Write an equation of a linear function in the form $y = mx + b$, given a graph, table, or a situation in context.
- Create a context for a linear function given a graph, table, or equation in the form $y = mx + b$.

8.PFA.4 The student will write and solve multistep linear equations in one variable, including problems in context that require the solution of a multistep linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

- Represent and solve multistep linear equations in one variable with the variable on one or both sides of the equation (up to four steps) using a variety of concrete materials and pictorial representations.
- Apply properties of real numbers and properties of equality to solve multistep linear equations in one variable (up to four steps). Coefficients and numeric terms will be rational. Equations may contain expressions that need to be expanded (using the distributive property) or require combining like terms to solve.

- c) Write a multistep linear equation in one variable to represent a verbal situation, including those in context.
- d) Create a verbal situation in context given a multistep linear equation in one variable.
- e) Solve problems in context that require the solution of a multistep linear equation.
- f) Interpret algebraic solutions in context to linear equations in one variable.
- g) Confirm algebraic solutions to linear equations in one variable.

8.PFA.5 The student will write and solve multistep linear inequalities in one variable, including problems in context that require the solution of a multistep linear inequality in one variable.

Students will demonstrate the following Knowledge and Skills:

- a) Apply properties of real numbers and properties of inequality to solve multistep linear inequalities (up to four steps) in one variable with the variable on one or both sides of the inequality. Coefficients and numeric terms will be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require combining like terms to solve.
- b) Represent solutions to inequalities algebraically and graphically using a number line.
- c) Write multistep linear inequalities in one variable to represent a verbal situation, including those in context.
- d) Create a verbal situation in context given a multistep linear inequality in one variable.
- e) Solve problems in context that require the solution of a multistep linear inequality in one variable.
- f) Identify a numerical value(s) that is part of the solution set of a given inequality.
- g) Interpret algebraic solutions in context to linear inequalities in one variable.